

## ANALYSIS OF AGE-CONCENTRATION PARAMETER CORRELATIONS IN GLOBULAR CLUSTERS BASED ON GAIA DR3

Sobir Turaev<sup>1</sup> , Qudratillo Yuldoshev<sup>1</sup> , Sirojiddin Turaev<sup>2</sup>  and Davron Rashidov<sup>3</sup> 

<sup>1</sup>*Ulugh Beg Astronomical Institute, Uzbekistan Academy of Sciences, Astronomy 33., Tashkent, 100052, Uzbekistan*  
E-mail: [sobr8488@mail.ru](mailto:sobr8488@mail.ru)

<sup>2</sup>*Karshi State Technical University, Mustakillik 225, 180100, Karshi, Uzbekistan*

<sup>3</sup>*National University of Uzbekistan named after Mirzo Ulugbek, University 4, Tashkent, 100174, Uzbekistan*

(Received: December 19, 2025; Accepted: May 8, 2026)

**SUMMARY:** This study investigates the empirical relationship between the ages and structural concentration parameters ( $\lambda$ ) of globular clusters (GCs). It underscores the cosmological role of GC ages as a lower limit for the Universe's age and a probe of early galactic assembly. The analysis critiques traditional age-dating challenges and leverages the superior astrometry of Gaia DR3 to compute  $\lambda$  for 106 Galactic GCs, expanding previous methodological work. We calculate concentration parameters of GCs using the Nuker profile fitting and analyze relationships with age determinations from major age catalogs (2008–2020). Using four modern age catalogs (Valcin et al. 2020, Dotter et al. 2010, Forbes & Bridges 2010, and VandenBerg et al. 2013), we find statistically significant inverse correlations ( $p < 0.05$ ) between  $\lambda$  and age, with Pearson coefficients  $r = -0.52$  to  $-0.78$  and Spearman coefficients  $r_s = -0.36$  to  $-0.59$ . The core finding is a moderate to strong inverse correlation ( $r \approx -0.52$  to  $-0.73$ ) between  $\lambda$  and age in most modern catalogs, suggesting that older clusters are more centrally concentrated. The correlation with dynamical age is weaker and not statistically significant ( $r_s = -0.15$ ,  $p = 0.233$ ). No significant correlation (0.30) is detected with the Koleva et al. (2008) data, potentially due to smaller sample size and larger standard deviation (2.27 Gyr). Our findings could provide robust constraints on galactic evolution scenarios and highlight the importance of homogeneous parameter analysis in the GC research. The correlation strength is method-dependent, reflecting systematic differences between catalogs. Furthermore, the synthesis indicates that the GC age likely serves as the dominant parameter governing horizontal branch morphology. The established link between cluster age and present-day structure could provide critical empirical constraints for models of stellar dynamics, galactic archaeology, and cosmological timelines.

**Key words.** Galaxy: globular clusters: general – Galaxy: evolution – Galaxy: structure – Stars: kinematics and dynamics – Astrometry – Methods: statistical

### 1. INTRODUCTION

GCs are the oldest known stellar systems in the Universe, and their derived ages provide a critical lower limit for the Universe's minimum age (Chaboyer et al. 1998, Freedman and Madore 2010,

Bennett et al. 2013). Beyond their cosmological significance, these ages are pivotal for studying the early Universe, as empirical correlations with other physical parameters offer key insights into evolutionary formation processes (Carretta et al. 2010, Gratton et al. 2012). The determination of GC ages is therefore fundamental to astrophysics, bridging the disciplines of stellar evolution, cosmology, and galactic archaeology. In stellar astrophysics specifically, GCs serve as unique natural laboratories because they comprise coeval populations of stars with a range

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of masses, providing a snapshot of stellar evolution (Renzini 2008).

However, the precise determination of their ages faces significant challenges. Systematic uncertainties persist due to degeneracies between fitted parameters such as age, metallicity, and helium abundance. Further complications arise from uncertainties in stellar physics, including convective core overshooting and elemental diffusion, as well as a reliance on an absolute age scale calibrated from only a limited sample of well-studied clusters (VandenBerg *et al.* 2014). These factors collectively represent the primary obstacles to achieving higher precision in the GC age dating.

Traditional approaches to the GC age determination have relied primarily on isochrone fitting of color-magnitude diagrams (VandenBerg *et al.* 2014, Dotter 2016). However, these methods face significant challenges including age-metallicity degeneracies, uncertainties in stellar physics, and limitations in absolute calibration (Chaboyer *et al.* 2022). The Gaia DR3 release addresses many of these limitations through improved parallax measurements (30% better precision than DR2), expanded radial velocity coverage (40 clusters vs. 15 in DR2), and enhanced photometric calibration (Gaia Collaboration *et al.* 2023, Lindegren *et al.* 2021).

The GC age determination has evolved through seminal contributions in three domains. Observational foundations were established by Sandage’s pioneering analysis of color-magnitude diagrams (Sandage 1954), while Chaboyer enabled cosmological applications by determining ancient ages (12–13 Gyr) that set a lower limit on the Universe’s age (Chaboyer *et al.* 1998). Theoretical modeling advanced through VandenBerg’s isochrones for metal-poor stars (VandenBerg *et al.* 2014) and Dotter’s stellar evolution libraries (Dotter *et al.* 2008). Observational precision was enhanced by Brown’s distance scale refinements (Freedman *et al.* 2001), Anderson’s HST (Hubble Space telescope) astrometry (Anderson and van der Marel 2010), and Vallenari’s Gaia-based recalibrations (Gaia Collaboration *et al.* 2018). Independent verification methods include Richer’s white dwarf cooling sequences (Hansen *et al.* 2007) and Bedin’s deep HST photometry (Bedin *et al.* 2009), providing crucial alternative age determinations. Piotto’s discovery of multiple populations fundamentally revised cluster formation models (Piotto *et al.* 2007). A modern synthesis of observational data, theoretical models, and independent validation techniques yields precise absolute ages for GCs, thereby refining our understanding of stellar evolution and cosmological timescales.

The correlation between GC ages and structural parameters represents a powerful probe of dynamical evolution over cosmological timescales. Early work by King (1966) established the theoretical foundation for understanding GC structure, while subsequent studies by Mocchi *et al.* (2013) expanded our empirical knowledge of cluster profiles. The advent

of large-scale surveys like Gaia has enabled systematic studies of GC populations across the Milky Way (Vasiliev and Baumgardt 2021).

Age-structural parameter correlations (Mackey & van den Bergh 2005) reveal how dynamical evolution drives increased core concentration in older clusters. The age-mass function relationship (de Marchi *et al.* 2007) demonstrates mass function flattening due to low-mass star depletion over time. Gaia DR3 has revolutionized this field by enabling precise, homogeneous parameter analysis (Vasiliev and Baumgardt 2021). These correlations test stellar evolution models (helium enrichment, mass loss) and constrain galaxy formation scenarios through accretion dating. Empirical age-parameter relationships remain crucial tools connecting stellar evolution with galactic archaeology and cosmology.

The analysis of GC surface density profiles to determine structural parameters was pioneered by King (1966), whose models provide the standard framework for fitting core, body, and halo profiles. Star-count studies revealed deviations from simple King models (described by Eq. 1), indicating extratidal stars (Grillmair *et al.* 1995). At large radii, star counts are essential for tracing outer profiles and distinguishing stellar populations, as demonstrated for post-core-collapse clusters (Noyola and Gebhardt 2006, Ferraro *et al.* 2003). Modern approaches like Baumgardt (2017) use N-body simulations to fit observed profiles, dynamically deriving masses and accounting for evolutionary effects such as tidal stripping and black hole retention.

The modern analysis of apparent surface density profiles to determine structural parameters of GCs was advanced by Nuritdinov *et al.* (2021), who used the HST data to calculate the concentration parameter ( $\lambda$ ) for 26 Galactic GCs. Building directly upon this methodology, our initial study (Turaev *et al.* 2024) extended this analysis to a sample of 81 clusters using data from Gaia DR2.

This work constitutes a significant expansion of those efforts. We analyze the apparent surface density profiles of 106 GCs from the superior Gaia DR3 archive, leveraging advanced methodological improvements. Our objectives are threefold, to compute the concentration parameter for this expanded sample, to conduct a robust analysis of the empirical relationship between  $\lambda$  and cluster age, and to investigate correlations between age with other fundamental physical parameters.

## 2. COMPARATIVE ANALYSIS OF GLOBULAR CLUSTER AGES FROM KEY CATALOGS

Table 1 summarizes the five age catalogs used in this study, including their methodologies, sample sizes, and statistical properties. A clear temporal progression is evident in the derived ages, with mean estimates increasing from  $11.1 \pm 0.4$  Gyr (Kol-

**Table 1:** Summary of age catalogs and correlation with  $\lambda$ .

Parameter	Koleva08	Dotter10	Forbes10	VandenBerg13	Valcin20
Method	Spectroscopy	HST/ACS	Literature	HST/ACS	Bayesian+Gaia
N (ages)	40	66	93	55	68
N (with $\lambda$ )	28	43	59	40	46
Mean age (Gyr)	11.09	11.94	12.03	12.57	12.76
Std. Dev. (Gyr)	2.27	0.92	1.33	1.23	1.19
Correlation $r$	0.30	-0.78	-0.53	-0.73	-0.52
p-value	0.12	< 0.001	<0.001	<0.001	<0.001

eva et al. 2008) to  $12.8 \pm 0.1$  Gyr (Valcin et al. 2020). Importantly, Table 1 also reports the correlation coefficient between  $\lambda$  and age for each catalog individually. The three catalogs with the highest precision (Dotter et al. (2010):  $\sigma = 0.92$  Gyr; Valcin et al. (2020):  $\sigma = 1.19$  Gyr; VandenBerg et al. (2013):  $\sigma = 1.23$  Gyr) all show statistically significant inverse correlations ( $r = -0.78$ ,  $-0.52$ , and  $-0.73$ , respectively, with  $p < 0.001$  for all). Forbes and Bridges (2010) ( $\sigma = 1.33$  Gyr) also shows a significant inverse correlation ( $r = -0.53$ ,  $p < 0.001$ ), while the catalog with the largest age uncertainty (Koleva et al. (2008):  $\sigma = 2.27$  Gyr) shows no significant correlation ( $r = 0.30$ ,  $p = 0.12$ ). This pattern suggests that the  $\lambda$ -age correlation is real but requires precise age measurements to be detected clearly. The sample sizes differ between the full catalogs and the number of clusters with available  $\lambda$  measurements (column 4 of Table 1). This is because surface density profiles could not be reliably fitted for all clusters due to poor convergence or physical plausibility of the derived parameters (e.g.,  $\lambda < 0$  or  $\lambda > 15$ ). A detailed description of the profile fitting procedure is given in Section 3. The mean age (row 4 of Table 1) is computed as an unweighted average of all available catalog values.

### 3. CALCULATING CONCENTRATION PARAMETER OF GLOBULAR CLUSTERS

The HST pioneered apparent surface density observations for 26 GCs, providing foundational data for structural analysis (Miocchi et al. 2013). Surface density profiles are derived by dividing the cluster into concentric annuli of equal area, establishing the relationship between stellar counts and the radial distance from the cluster center.

GCs typically exhibit centrally peaked stellar concentrations, with each cluster displaying a unique concentration gradient. Nuritdinov et al. (2021) introduced a generalized concentration parameter to quantify this central concentration using a modified King model (King 1966):

$$\sigma(\sigma_0, \lambda, r_*) = \sigma_0 \left(1 + \frac{r^2}{r_*^2}\right)^{-\lambda} \quad (1)$$

where  $\lambda$  is the concentration parameter (represents the increasing rate of stellar concentration towards the center),  $r_*$  is the scale radius marking the transi-

tion to a steeper profile, and  $\sigma_0$  is the central surface density. The authors applied simplex minimization (described by Eq. 2) to the HST data for 26 clusters<sup>1</sup> to determine structural parameters:

$$F(\sigma_0, \lambda, r_*) = \sum_k [\sigma(r, \sigma_0, \lambda, r_*) - \sigma_{\text{obs}}^k]^2 \quad (2)$$

Subsequently, they validated these results using the  $\chi^2$  minimization (Nuritdinov et al. 2022):

$$\chi^2 = \sum_k \frac{[\sigma(r, \sigma_0, \lambda, r_*) - \sigma_{\text{obs}}^k]^2}{\sigma(r, \sigma_0, \lambda, r_*)} \quad (3)$$

The empirical relationship between concentration parameters and cluster ages for 22 of these clusters was subsequently analyzed by Turaev and Rashidov (2025).

The Gaia DR2 project significantly expanded this data set, providing surface density profiles for 81 clusters (de Boer et al. 2019). In Turaev et al. (2024), the Nuker model (described by Eq. 4) was identified as the optimal fit for Gaia DR2 surface density data:

$$\sigma(r, \lambda_1, \lambda_2, r_*, \sigma_0) = \sigma_0 \left(\frac{r}{r_*}\right)^{-\lambda_1} \left(1 + \frac{r}{r_*}\right)^{-\lambda_2}. \quad (4)$$

Here,  $\lambda_1$  and  $\lambda_2$  are concentration parameters for the inner and outer parts of the surface density profiles, respectively. In this case,  $\lambda_1 + \lambda_2 = \lambda$  is the concentration parameter for the whole profile. The values of this parameter are given in column 2 of Table 2. For comparison, the traditional King (1966) concentration parameter  $c = \log(r_t/r_c)$  measures the ratio of tidal to core radius. Unlike  $c$ , which depends on the outer truncation radius, the Nuker-based  $\lambda(\lambda_1 + \lambda_2)$  characterizes the asymptotic slope of the outer density profile in the halo region. Our comparison reveals a weak, statistically non-significant negative correlation (Spearman  $\rho = -0.16$ ,  $p = 0.23$ ), indicating that while both parameters relate to cluster structure, they capture distinct physical properties. The King (1966)  $c$  is sensitive to the core-to-tidal size ratio, whereas  $\lambda$  primarily describes the steepness of the outer envelope. Thus,  $\lambda$  offers complementary information about the outer dynamical state, particularly for clusters undergoing tidal disruption. The structural parameters of the Model (described by Eq. 4) were calculated using the simplex minimiza-

<sup>1</sup><http://www.cosmic-lab.eu/catalog/>

tion method. In addition to the structural parameters, we incorporate a proxy for dynamical evolution by introducing the dynamical age of the cluster. The dynamical age is defined as the number of elapsed half-mass relaxation times:

$$\tau_{\text{dyn}} = \frac{\text{Age}}{t_{\text{rh}}} \quad (5)$$

where  $t_{\text{rh}}$  is the half-mass relaxation time taken from the Harris (2010) catalog. This parameter provides a physically motivated measure of cluster dynamical evolution, complementing the chronological age used throughout this study.

Subsequently, the observational dataset was expanded through the Gaia DR3 project, increasing the number of GCs with measured surface brightness profiles to  $106^2$ . Based on these updated observations, we calculate the concentration parameter ( $\lambda$ ) for newly observed GCs using model (described by Eq. (4)) and analyze its relationship with age of the cluster. Furthermore, since the age values for the clusters in the key references in this paper were derived using different methodologies and consequently vary from one another, we identify and conduct a comparative analysis of the empirical relationships between the concentration parameter and the age values specifically derived for each individual reference study.

## 4. RELATIONSHIP BETWEEN AGE AND PHYSICAL PARAMETERS OF GLOBULAR CLUSTERS

### 4.1. Dominant Parameters in Globular Cluster Evolution

A synthesis of key studies establishes that age is the dominant second parameter governing GC properties after metallicity. Strong correlations exist between the age and horizontal branch (HB) morphology, metallicity, and Galactic position, revealing the Milky Way’s assembly history. The long-standing “second parameter problem”, where metallicity alone cannot explain the HB morphology diversity, finds its solution in cluster age. Dotter *et al.* (2010), using homogeneous HST/ACS photometry of 60 GCs, demonstrated that after removing metallicity effects, the correlation between  $\Delta(V - I)$  (HB morphology indicator) and age is stronger than that of any other parameter. They conclusively identified age as the second parameter, with central density as a likely third parameter.

Marín-Franch *et al.* (2009) and De Angeli *et al.* (2005) found that metal-poor clusters ( $[\text{Fe}/\text{H}] < -1.7$ ) are old and coeval, with an age dispersion consistent with zero. In contrast, intermediate-metallicity clusters ( $-1.7 < [\text{Fe}/\text{H}] < -0.8$ ) are, on average, 1.5 Gyr younger, showing significant age dispersion. This bifurcation points to an extended, multi-phase formation history for the Galaxy. Evidence from massive

clusters suggests the internal cluster dynamics, likely tied to the central density, acts as a third parameter influencing detailed chemical signatures.

Carretta *et al.* (2010) demonstrated that massive GCs like M54 and  $\omega$  Centauri exhibit intrinsic metallicity dispersions and complex light-element anti-correlations (e.g., Na-O, Mg-Al). Crucially, the most extreme chemical anomalies are found in the metal-rich components of these clusters, indicating that star formation for this population was delayed by 10–30 Myr. This requires a dense environment to retain processed gas from a first generation of stars.

GC ages and metallicities show a bifurcated relationship tracing different formation pathways. Forbes and Bridges (2010) analyzed 93 GCs, finding two distinct tracks: an ancient, metal-poor population ( $\sim 12.8$  Gyr) and a younger population dominated by accreted clusters from dwarf galaxies such as Sagittarius and Canis Major. They estimated that 25% of the Galactic GC system was accreted, with accreted clusters showing distinct chemical enrichment histories.

The oldest GCs provide a lower limit to the Universe’s age. Valcin *et al.* (2020) determined a mean age of  $13.32 \pm 0.10$  Gyr for the most metal-poor GCs using Bayesian analysis of 68 clusters, inferring a Universe age of  $t_U = 13.5$  Gyr. This independently confirms the cosmological constraints from Planck CMB measurements (Planck Collaboration *et al.* 2020).

VandenBerg *et al.* (2013) established central density as a critical third parameter in GC evolution, linking high density to distinct subgiant morphologies and a higher incidence of multiple stellar populations, likely from retained gas for second-generation stars. This reveals a hierarchical framework: metallicity is the primary parameter, age controls horizontal branch morphology, and central density regulates multiple population formation. These correlations, supported by the HST photometry and spectroscopy, form an empirical basis for Galactic evolution models.

### 4.2. Age - Concentration Parameter Relationship

A discrepancy exists between the sample sizes of clusters with determined concentration parameters versus those with age values across literature sources. For instance, in Valcin *et al.* (2020), age values are provided for 112 clusters, while concentration parameter values are calculated for 46 of these clusters. In Dotter *et al.* (2010), age values are given for 66 clusters, with concentration values available for 43 of them. In VandenBerg *et al.* (2013), among 55 clusters with age determinations, concentration parameters are calculated for 41 clusters. Forbes and Bridges (2010) provides age values for 93 clusters, with concentration values determined for 59 clusters. In Kolva *et al.* (2008), concentration parameters are calculated for 28 out of 40 clusters with available age data. Although surface density profiles were constructed for

<sup>2</sup><https://gea.esac.esa.int/archive/>

**Table 2:** Concentration parameter ( $\lambda$ ) and age values of GCs compiled from key catalogs.

GC Name	$\lambda$	Valcin20	Dotter10	Forbes10	VandenBerg13	Koleva08	Mean Age (Gyr)
Arp 2	3.3	13.42	13	10.88	12	-	12.33
IC 4499	5.7	12.8	-	-	-	-	12.80
E 3	3.43	-	-	12.8	-	-	12.80
NGC 104	3.01	13.54	12.75	13.06	11.75	10.7	12.36
NGC 288	5.8	11.2	12.5	10.62	11.5	-	11.46
NGC 1261	5.38	-	11.5	10.24	10.75	-	10.83
NGC 1904	3.71	-	-	11.14	-	11.7	11.42
NGC 2298	4.84	13.89	13	12.67	-	12.6	13.04
NGC 2419	3.84	-	13	12.3	-	-	12.65
NGC 2808	3.99	10.93	-	10.8	11	10.2	10.73
NGC 3201	3.41	13.05	12	10.24	11.5	11.3	11.62
NGC 4147	1.7	13.02	12.75	11.39	12.25	-	12.35
NGC 4590	5.12	12.03	13	11.52	12	-	12.14
NGC 4833	1.2	14.69	13	12.54	12.5	-	13.18
NGC 5024	3.27	12.11	13.25	12.67	12.25	-	12.87
NGC 5139	6.42	14.91	-	11.52	-	-	13.22
NGC 5272	3.59	12.6	12.5	11.52	11.75	-	12.09
NGC 5466	2.88	12.31	13	13.57	12.5	-	12.85
NGC 5634	3.29	-	-	11.84	-	-	11.84
NGC 5694	2.48	-	-	13.44	-	-	13.44
NGC 5824	4.27	-	-	12.8	-	-	12.80
NGC 5904	3.6	12.75	12.25	10.62	11.5	10.9	11.60
NGC 6093	3.24	13.83	13.5	12.54	-	-	13.29
NGC 6121	5.48	13.01	12.5	12.54	11.5	11.7	12.25
NGC 6144	1.96	14.47	13.5	13.82	12.75	-	13.64
NGC 6171	4.33	-	12.75	13.95	12	11.7	12.60
NGC 6205	4.1	13.49	13	11.65	12	-	12.54
NGC 6235	3.71	12.85	13	11.39	-	12	12.31
NGC 6254	5.61	-	-	11.39	11.75	11.8	11.65
NGC 6266	2.07	-	-	11.14	-	12	11.89
NGC 6273	2.95	-	-	11.9	-	-	11.90
NGC 6284	2.01	-	-	11.14	-	12	11.57
NGC 6287	5.04	-	-	13.57	-	-	13.57
NGC 6316	-	-	-	-	-	10	10.00
NGC 6341	1.55	13.3	13.25	13.18	12.75	-	13.12
NGC 6342	2.2	-	-	12.03	-	12	12.02
NGC 6366	7.15	12.15	12	13.31	11	-	12.12
NGC 6388	2.38	11.07	-	12.03	-	10.6	11.23
NGC 6397	0.93	14.21	13.5	12.67	13	-	13.35
NGC 6426	1.76	13.92	-	12.9	-	-	13.41
NGC 6441	4.13	10.44	-	11.26	-	12.7	12.08
NGC 6496	6.37	10.86	12	12.42	10.75	-	11.51
NGC 6541	2.21	13.51	13.25	12.93	12.5	-	13.05
NGC 6569	4.47	-	-	-	-	10.9	10.90
NGC 6584	1.13	12.72	12.25	11.26	11.75	-	12.00
NGC 6624	1.28	11.26	13	12.54	11.25	10.6	11.73
NGC 6626	3.12	-	-	-	-	12	12.00
NGC 6637	3.74	12.95	12.5	13.06	11	10.6	12.02
NGC 6638	5.92	-	-	-	-	11.5	11.50
NGC 6652	1.89	12.98	13.25	12.93	11.25	11.4	12.36
NGC 6681	1.25	13.87	13	12.8	12.75	-	13.11

**Table 3:** Concentration parameter ( $\lambda$ ) and ages of GCs from key catalogs (continued from Table 2).

GC Name	$\lambda$	Valcin20	Dotter10	Forbes10	VandenBerg13	Koleva08	Mean Age (Gyr)
NGC 6715	4.58	12.22	-	-	11.75	-	11.99
NGC 6717	2.46	11.65	13	13.18	12.5	-	12.58
NGC 6723	3.34	13.81	12.75	13.06	12.5	11.6	12.74
NGC 6752	2.24	13.48	12.5	11.78	12.5	12.2	12.49
NGC 6779	3.14	14.85	13.5	13.7	12.75	-	13.70
NGC 6809	4.32	13.93	13.5	12.29	13	-	13.18
NGC 6864	12.03	-	9.98	-	-	-	9.98
NGC 6934	5.05	13.24	12	11.14	11.75	-	12.03
NGC 6981	3.24	12.72	12.75	10.88	11.5	-	11.96
NGC 7078	1.43	13.28	13.25	12.93	12.75	12	12.84
NGC 7089	2.91	13.08	12.5	11.78	11.75	-	12.28
NGC 7099	1.38	12.82	13.25	12.93	13	-	13.00
NGC 7492	4.76	-	-	12	-	-	12.00
Pal 12	10.42	9.94	9.5	8.83	9	-	9.32
Terzan 7	8.53	8.1	8	7.3	-	-	7.80

106 Galactic GCs using Gaia DR3 data, only 65 clusters were included in the correlation analysis. Age values are not available for all clusters. The number of clusters where age values (available in the key literature) and  $\lambda$  values intersect is 65. Of course,  $\lambda$  values are not calculated for all clusters, clusters with slopes  $\lambda < 0$  or  $\lambda > 15$  that do not have physical meaning, e.g. Ruprecht 106, Pyxis, E3 (but age values for these clusters are not available in the key literature).

We separately calculate the correlation coefficients between the concentration parameter and the age provided in each of the aforementioned literature sources. Additionally, we examine the correlation between the concentration parameter and the arithmetic mean of age values available across all literature sources for each cluster. To assess the physical origin of the observed correlations, we repeat the analysis using dynamical age ( $\tau_{\text{dyn}}$ ) as an alternative independent variable. This allows us to test whether the concentration parameter is more strongly linked to dynamical evolution rather than absolute formation time. A comparison between chronological and dynamical age correlations is presented in Table 4.

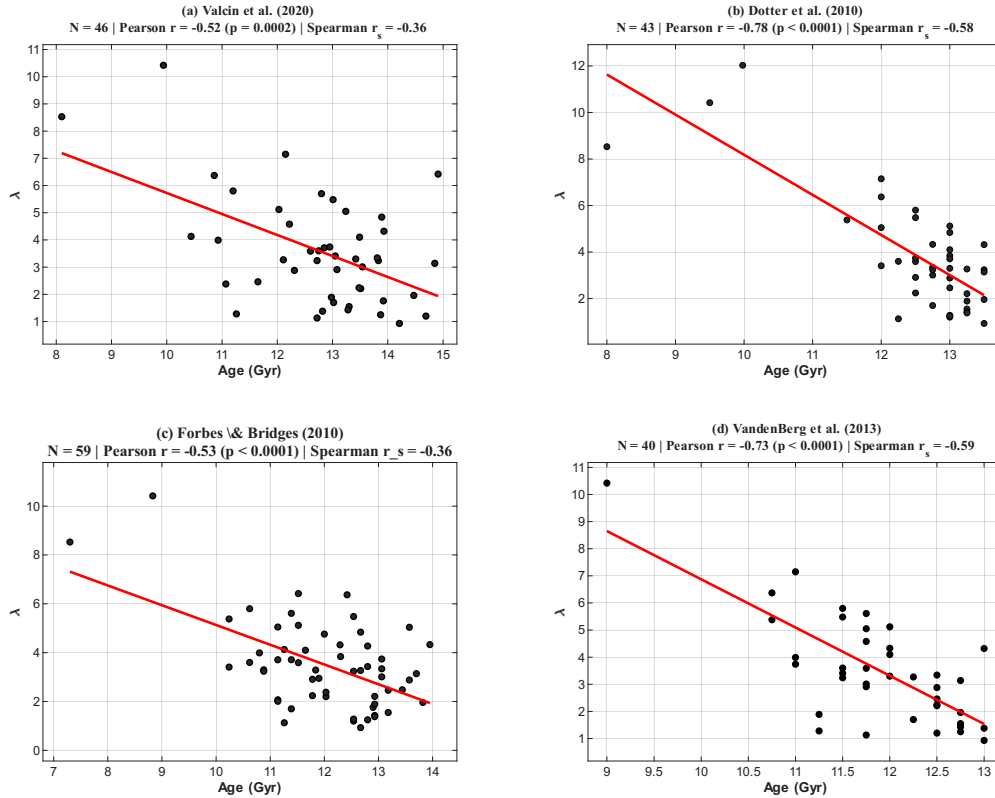
$$\lambda = (\alpha \pm \Delta\alpha) \tau + (\beta \pm \Delta\beta) \quad (6)$$

The linear empirical formulas between the age  $\tau$  and the concentration parameters  $\lambda$  are determined according to Eq. (6), and the obtained correlation coefficients and the coefficients of the empirical formula ( $\alpha$  and  $\beta$  with standard deviations) between the parameters are provided in Table 5. Fig. 1 displays the relationship diagrams between stellar population ages and concentration parameters for GCs, as derived from individual literature sources. The observed scatter in these diagrams reflects a combination of measurement uncertainties in both age and  $\lambda$ , as well as intrinsic differences in cluster evolutionary histories, including variations in relaxation times, orbital parameters, and tidal interactions.

It can be seen from this table that there are good negative correlations with the age values from VandenBerg *et al.* (2013) and Dotter *et al.* (2010) (specifically -0.73 and -0.72, respectively). Relatively weak negative correlations exist with age values corresponding to Valcin *et al.* (2020), Forbes and Bridges (2010), and Mean age (specifically -0.52, -0.53, and -0.55, respectively). Unfortunately, no correlation was found between the concentration parameter and the age values provided by Koleva *et al.* (2008). This situation may be attributed to the relatively small number of clusters, the relatively large standard deviation (2.27), and the uncertainties in the initial analysis methods.

The use of multiple age catalogs introduces potential systematic differences due to varying methodologies. However, this approach allows us to test the robustness of the inferred correlations across independent datasets. We find that the inverse relationship between  $\lambda$  and age persists when restricting the analysis to more recent and homogeneous catalogs, indicating that the observed trend is not driven by systematic biases in any single study. To ensure statistical robustness, we complement the Pearson correlation analysis with Spearman rank correlation coefficients and corresponding p-values. The Spearman test is less sensitive to outliers and non-linear trends, providing a more reliable assessment of monotonic relationships.

We note that the Spearman rank correlation coefficients are systematically lower than the Pearson coefficients (Table 5), particularly for the Forbes and Bridges (2010) catalogs. This suggests that the relationship between  $\lambda$  and age is approximately linear but with some scatter, and that the correlation is not driven by outliers. The consistency of the sign (negative) and statistical significance ( $p < 0.05$ ) across all four independent catalogs confirms the robustness of the observed inverse correlation.



**Fig. 1:** Diagrams of age-concentration parameter relationships for GCs from key references.

**Table 4:** Comparison of correlations with chronological and dynamical age.

Variable	N	Spearman $r_s$	p-value	Pearson $r$
Chronological Age (mean)	65	-0.58	<0.001	-0.55
Dynamical Age ( $N_{relax} = Age/t_{rh}$ )	65	-0.15	0.233	-0.12

**Table 5:** Correlation between the concentration parameter  $\lambda$  and cluster ages from individual catalogs. Pearson correlation coefficients ( $r$ ) with corresponding linear regression parameters ( $\alpha$ ,  $\beta$ ) and Spearman rank correlation coefficients ( $r_s$ ) with p-values.

Catalog	N	Pearson $r$	p-value	$\alpha$ (slope)	$\beta$ (intercept)	Spearman $r_s$	p-value
Valcin et al. (2020)	46	-0.52	<0.001	$-0.34 \pm 0.09$	$14.02 \pm 0.35$	-0.36	0.013
Dotter et al. (2010)	43	-0.78	<0.001	$-0.36 \pm 0.06$	$13.96 \pm 0.23$	-0.58	<0.001
Forbes & Bridges (2010)	59	-0.53	<0.001	$-0.30 \pm 0.06$	$13.12 \pm 0.27$	-0.36	0.006
VandenBerg et al. (2013)	40	-0.73	<0.001	$-0.30 \pm 0.05$	$12.94 \pm 0.18$	-0.59	<0.001

To assess whether the scatter in Fig. 1 is influenced by environmental effects, we performed a basic sanity check by examining the correlation between  $\lambda$  and Galactocentric distance ( $R_{GC}$ ), using values from the Harris (2010) catalog. We find no statistically significant correlation (Spearman  $\rho = 0.09$ ,  $p = 0.47$ ), suggesting that  $R_{GC}$  is not a primary driver of the observed scatter within the present sample. Nevertheless, we caution that  $R_{GC}$  represents only a snapshot of the current Galactic environment. A more complete treatment including orbital param-

eters (e.g., perigalactic distances and orbital eccentricities) would be required to fully disentangle the role of tidal interactions.

## 5. CONCLUSION

This study systematically investigated the correlation between the structural concentration parameter ( $\lambda$ ) and the ages of Galactic GCs, utilizing an expanded sample of 106 clusters and comparing five major age catalogs spanning 2008–2020. Our analysis reveals a moderate to strong inverse correlation

( $r \approx -0.52$  to  $-0.73$ ) between  $\lambda$  and age in the majority of modern catalogs. This robust empirical relationship indicates that older GCs tend to exhibit higher central concentration — a result consistent with long-term dynamical evolution where processes such as two-body relaxation and mass segregation drive an increase in central density over cosmic time.

### 5.1. Implications for Galactic Assembly Scenarios

The observed inverse correlation between  $\lambda$  and age has direct implications for Galactic assembly scenarios. The oldest, most concentrated clusters (e.g., NGC 6341 with  $\lambda = 1.55$ , age 13.1 Gyr) likely formed during the early, violent collapse phase of the Milky Way, when high ambient gas densities promoted efficient core concentration. In contrast, younger, less concentrated clusters (e.g., Pal 12 with  $\lambda = 10.42$ , age 9.3 Gyr) show lower concentration, consistent with their formation in dwarf galaxies that were later accreted by the Milky Way. This bifurcation in  $\lambda$ -age space supports the dual formation channel proposed by Forbes and Bridges (2010): an *in situ* population of ancient, concentrated clusters and an accreted population of younger, less concentrated clusters. The wide scatter in  $\lambda$  among intermediate-age clusters (10–12 Gyr) may reflect the diversity of accretion events that assembled the Milky Way’s halo. We find that the correlation between  $\lambda$  and dynamical age is weaker and statistically not significant compared to that with chronological age (Table 4).

The weak and statistically non-significant correlation between  $\lambda$  and dynamical age (Spearman  $r_s = -0.15$ ,  $p = 0.233$ ) merits discussion. This finding suggests that the concentration parameter  $\lambda$  is more sensitive to initial formation conditions rather than the subsequent dynamical evolution time expressed in units of relaxation time. Several factors may contribute to this result. First, half-mass relaxation times ( $t_{rh}$ ) depend on cluster mass and half-mass radius, parameters that themselves correlate with concentration. This introduces a degree of circularity when constructing the dynamical age proxy. Second, the wide range of relaxation time estimates (spanning approximately two orders of magnitude) may wash out any underlying correlation. Third, clusters of the same chronological age can be at different dynamical stages depending on their mass, orbital history, and tidal environment. Therefore, while dynamical age is theoretically the more relevant parameter for quantifying evolutionary progress, chronological age proves to be a more practical empirical tracer of structural concentration for Galactic GCs with the current data quality. These results should be interpreted as empirical correlations rather than direct evidence of dynamical evolution.

The absence of a significant correlation with the Koleva et al. (2008) catalog ( $r = 0.30$ ) likely stems from its comparatively smaller sample size and larger intrinsic scatter (standard deviation of 2.27 Gyr com-

pared to 0.92–1.33 Gyr for more recent catalogs), underscoring the substantial impact of systematic uncertainties and methodological homogeneity on derived empirical relationships. These core findings highlight a hierarchical parameter framework governing GC evolution: metallicity as the primary driver, age as the dominant second parameter shaping horizontal branch morphology, and central density — probed here by  $\lambda$  — acting as a key third parameter regulating internal dynamics and the formation of multiple populations.

### 5.2. Prospects for Future Surveys

Future large-scale surveys will dramatically improve our ability to refine these correlations. Gaia DR4, expected to deliver proper motions for an order of magnitude more stars, will enable surface density profile construction for nearly all known Galactic GCs, expanding our sample from 106 to approximately 150 clusters. The Vera C. Rubin Observatory’s Legacy Survey of Space and Time (LSST) will provide deep, multi-epoch photometry reaching the main-sequence turnoff even in the most distant GCs, enabling homogeneous age determination for clusters currently lacking precise age measurements. Furthermore, the combination of structural parameters from Gaia and chemical abundances from upcoming spectroscopic surveys (e.g., WEAVE, 4MOST) will allow us to disentangle the competing effects of age, metallicity, and dynamics on the present-day structure of GCs. These upcoming datasets will enable a transition from empirical correlations to a physically motivated predictive framework.

Our results provide critical empirical constraints for models of star cluster dynamics, galactic archaeology, and cosmological timelines, reaffirming GCs as indispensable probes of early-Universe conditions and galaxy assembly. The concentration parameter  $\lambda$  emerges as a powerful, complementary tool for tracing cluster evolutionary states, particularly when combined with precise age determinations. Future work combining larger astrometric samples with detailed chemical abundance measurements will further refine these correlations and their physical interpretations.

*Acknowledgements* – This research was conducted within the framework of the Grant FZ-2020092851 funded by the Agency for Innovative Development under the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan. We are grateful to the anonymous referee for careful reading of the manuscript and for valuable suggestions that helped to improve the quality of this work.

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**АНАЛИЗА КОРЕЛАЦИЈА ИЗМЕЂУ СТАРОСТИ И ПАРАМЕТРА  
КОНЦЕНТРАЦИЈЕ КОД ГЛОБУЛАРНИХ ЈАТА НА ОСНОВУ  
GAIA DR3 КАТАЛОГА**

Sobir Turaev<sup>1</sup> , Qudratillo Yuldoshev<sup>1</sup> , Sirojiddin Turaev<sup>2</sup>  and Davron Rashidov<sup>3</sup> 

<sup>1</sup> *Ulugh Beg Astronomical Institute, Uzbekistan Academy of Sciences, Astronomy 33., Tashkent, 100052, Uzbekistan*

E-mail: [sobr8488@mail.ru](mailto:sobr8488@mail.ru)

<sup>2</sup> *Karshi State Technical University, Mustakillik 225, 180100, Karshi, Uzbekistan*

<sup>3</sup> *National University of Uzbekistan named after Mirzo Ulugbek, University 4, Tashkent, 100174, Uzbekistan*

УДК 524.4–327 : 524.6 : 524.8

*Оригинални научни рад*

Ова студија истражује емпиријску везу између старости и структурних параметара концентрације ( $\lambda$ ) глобуларних јата (енг. *globular clusters*, GCs). Посебно се истиче космолошка улога старости глобуларних јата као доње границе старости Универзума и показатеља раног формирања галаксија. Анализа указује на ограничења традиционалних метода одређивања старости и користи предности високо прецизне астрометрије Gaia DR3 каталога за израчунавање параметра  $\lambda$  за 106 галактичких глобуларних јата, чиме се проширују претходна методолошка истраживања. Параметри концентрације глобуларних јата израчунати су фитовањем Нукер (Nuker) профилем, а затим су анализирани њихове везе са проценама старости из главних каталога старости објављених у периоду 2008–2020. Коришћењем четири савремена каталога старости (Valcin et al. 2020, Dotter et al. 2010, Forbes & Bridges 2010 и VandenBerg et al. 2013), добијене су статистички значајне инверзне корелације ( $p < 0,05$ ) између  $\lambda$  и старости, са Пирсоновим коефицијентима  $r = -0,52$  до  $-0,78$  и Спирмановим коефицијентима  $r_s = -0,36$  до  $-0,59$ . Главни

резултат рада је пронађена умерена до јака инверзна корелација ( $r \approx -0,52$  до  $-0,73$ ) између  $\lambda$  и старости у већини савремених каталога, што указује на то да су старија јата више централно концентрисана. Корелација са динамичком старошћу је слабија и није статистички значајна ( $r_s = -0,15$ ,  $p = 0,233$ ). Није уочена значајна корелација ( $0,30$ ) са подацима из рада Koleva et al. (2008), што је могуће последица мањег узорка и веће стандардне девијације ( $2,27$  Gyr). Добијени резултати могу пружити значајна ограничења за сценарије галактичке еволуције и указују на важност хомогене анализе параметара у истраживању глобуларних јата. Јачина корелације зависи од примењене методе, и одражава систематске разлике између каталога. Поред тога, синтеза резултата кључних студија указује да старост глобуларних јата вероватно представља доминантан параметар који одређује морфологију хоризонталне гране. Утврђена веза између старости јата и њихове садашње структуре може пружити важна емпиријска ограничења за моделе звездане динамике, галактичке археологије и космолошке временске оквире.