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DARK MATTER IN MASSIVE EARLY-TYPE GALAXIES: 100 YEARS OF THE JEANS EQUATIONS

S. Samurović

Astronomical Observatory, Volgina 7, 11060 Belgrade 74, Serbia
E–mail: *srdjan@aob.rs*

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SUMMARY: In 1919 James Jeans published the book *Problems of Cosmogony and Stellar Dynamics* **in which he summarized his work on dynamics of stellar systems based on his papers published from 1915 onwards. To mark the centenary of this publication we present here one application of his work relevant for contemporary research of galaxies: we analyze the problem of dark matter in massive early-type galaxies (ellipticals and lenticulars) using various available observational data. After discussing the basics of the Jeans equations we study their application on the integrated stellar spectra of galaxies, planetary nebula data and, especially, globular cluster data. We rely on both Newtonian and MOND frameworks and show their advantages and drawbacks. To infer the contribution of the dark component in early-type galaxies we rely on several stellar population synthesis models. It is shown that dark matter does not dominate in the inner regions of early-type galaxies, but becomes more important beyond three effective radii.**

Key words. Galaxies: kinematics and dynamics – Galaxies: elliptical and lenticular, cD – Galaxies: structure – Dark matter

1. INTRODUCTION

Although the problem of dark matter (DM) and its contribution to the total dynamical mass of all morphological types of galaxies is present since 1970's, one should expect that the solution would have been found, or at least that the obstacles to overcome would be well-defined. Instead, not only is the explanation not in sight, but the DM problem itself has become even more complex. New observations did not bring us closer to solving the issue of DM, but instead posed new questions which need to be properly answered if we are to fully understand and finally solve the problem of the measured dynamical masses of galaxies of various morphological (Hubble) types.

The large quantities of DM in spiral galaxies are well documented (see e.g., Bertin 2014): the studies of their rotation curves (RCs) which provide rotation velocity as a function of the disk radius and which remain constant at large galactocentric distances (see e.g., Sofue and Rubin 2001) provided a plethora of evidence for this. The available data such as the THINGS[1] (The HI Nearby Galaxy Survey) database (Walter et al. 2008) allow for various studies of DM in nearby spirals: for example, Jovanović (2017) analyzed the spiral galaxy NGC 5055, and also dwarf irregular galaxy DDO 154, using THINGS to examine their RCs and 3.6 μm images from the Spitzer Survey of Stellar Structure in Galaxies[2] (S4G, Sheth et al. 2010) database to obtain the dynamical mass of the stellar component.

[1] `http://www.mpia.de/THINGS`

[2] `http://irsa.ipac.caltech.edu/data/SPITZER/SINGS/galaxies`

The RCs of both galaxies are well-fitted with models with significant DM contribution. The recent work by Genzel et al. (2017) in which the RCs of the outer disks of six massive star-forming galaxies at redshifts z between approximately 0.6 and 2.6 were analyzed showed that the RCs decrease with radius which may imply the lack of DM beyond the local Universe.

We presented the most important details regarding the study of DM in early-type galaxies (hereafter, ETGs) which include ellipticals and lenticulars in Samurović (2007) and here we intend to provide the update and to describe the most important new observational discoveries and theoretical achievements published since then. One should bear in mind that ETGs are studied to a lesser degree than their spiral counterparts for several reasons. First, there is a lack of cool gas (neutral hydrogen HI) in ETGs which is necessary for obtaining RCs at large galactocentric distances. The observational situation improved with the results obtained using the HI Parkes All-Sky Survey (HIPASS)[3], the large collection of high-resolution HI data of a homogeneously selected sample (see Oosterloo et al. 2007). Also, ETGs are faint in their outer parts and thus long exposures are needed to obtain high quality spectra and/or photometric data at large galactocentric distances (see e.g., Spolaor et al. 2010). Another problem with analysis of the mass content in ETGs is the problem of knowledge of the shape of orbits in these galaxies which leads to the mass-anisotropy degeneracy.

The contemporary studies of the DM in ETGs rely on the following types of studies (see e.g., Samurović 2014, Bertin 2014): (i) long-slit spectra and integral field spectroscopy data are used to study the contribution of the visible (i.e. stellar) component to the total dynamical mass, (ii) study of the kinematics of various tracers of the total dynamical mass such as planetary nebulae (PNe) and globular clusters (GCs), (iii) study of the X-rays in ETGs and (iv) analysis of gravitation lensing in ETGs. Also, stellar shells can be used to investigate the gravitational fields of ETGs at large galactocentric radii (Bílek et al. 2015, 2016).

Since the beginning of the DM studies in ETGs solving of the Jeans equations became one of the most important tools and this will be described below in more details. However, it is important to stress here that the approach based on the Jeans equations is not the only viable technique to estimate DM in massive ETGs. There exist arguably more sophisticated methods which include the method based on orbit superposition (Schwarzschild 1979) and the made-to-measure modeling based on the distribution functions (the algorithm of Syer and Tremaine 1996 extended in the NMAGIC code, see de Lorenzi et al. 2007). These approaches are shown to be useful, but they are much more computationally demanding. However, the mass estimates based on them are expected to be consistent with the results obtained using the Jeans models, see e.g. Deason et al. (2012) who warn: "it is not obvious whether this extra complication makes any appreciable difference to the mass estimates".

In the course of analysis of the DM problem in ETGs, in analogy with spirals which are known for large amounts of DM, various attempts to apply some alternative theory of gravitation such as the MOdified Newtonian Dynamics (MOND; Milgrom 1983) on ETGs appeared in the literature. Since MOND was well tested on spirals and proved to be successful in such objects, one could expect that in ETGs the application of MOND should also be useful. However, this was not always the case as will be discussed below. For example, Durazo et al. (2017) analyzed the dynamics of GCs and elliptical galaxies in MOND and showed that a universal projected velocity dispersion profile accurately describes various classes of pressure supported systems, and that the expectations of MOND are satisfied across seven orders of magnitude in mass, but the median of velocity dispersions of the ellipticals tested was 116 km s^{-1} and the data extended out to $\sim 3R_{\rm e}$, where $R_{\rm e}$ is the effective radius, i.e., the radius which encompasses half of the total light of the given galaxy. Famaey and McGaugh (2012) in their review discussed pressure-supported stellar systems (such as ETGs) in MOND and conclude that "the successes of MOND are in general a bit less impressive than in rotationally-supported ones". Subsequent studies only strengthened their conclusion as will be discussed below.

The plan of the paper is as follows. In Section 2 we present the basics of the Jeans equations which are necessary to understand the analysis in the remaining chapters. In Section 3 we analyze the application of the Jeans equations using integrated stellar spectra in galaxies. In Section 4 we analyze various other tracers of gravitational potential in ETGs such as PNe and GCs. Finally, in Section 5 we discuss the main conclusions obtained using the Jeans equations and their implications for future investigation of DM in elliptical and lenticular galaxies.

2. THEORETICAL AND OBSERVATIONAL FOUNDATIONS OF THE JEANS EQUATIONS

The Jeans equations were introduced by Sir James Jeans in a series of papers (Jeans 1915, 1916a, 1916b) and later in his book *Problems of Cosmogony and Stellar Dynamics* (Jeans 1919) in which he summed up his research. More precisely, it was Jeans who for the first time applied the equations related to the equilibria of the collisionless systems to stellar systems. These equations were derived previously by James Clerk Maxwell (Maxwell 1867) but since he already had a set of equations named after him, and it was Jeans who first applied them to stellar systems, the equations in question are called the Jeans equations. On page 229 of his book Jeans

[3] `http://www.atnf.csiro.au/research/multibeam/release`

says: "It now appears that, for our present universe[4], the problem of stellar dynamics is the same as the problem of the kinetic theory of gases with the collisions left out. This being so, stellar dynamics is naturally very much simpler than gas-dynamics". In the literature one can find several different designations used for the equation shown below (Eq. 1). Hénon (1982) lists seven names: Liouville equation, Boltzmann equation, collisionless Boltzmann equation, Liouville-Boltzmann equation, Jeans equation, equation of continuity and Vlasov equation. The reader is referred to Hénon (1982) for a discussion of the history of the work done to deduce this equation; we note that he suggests that the proper name would be "collisionless Boltzmann equation". Robson et al. (2017) on the 150th anniversary of the Maxwell's paper in their review of the Maxwell-Boltzmann formalism call the equations devised by Maxwell in 1867 "Maxwell's (other) equations" to distinguish them from his fundamental work related to electromagnetism.

The mathematical foundations of the Jeans equations are given in Binney and Tremaine (2008, hereafter BT08). Below, the most important steps related to their derivation are presented which will make their application to ETGs clear and justified.

We first assume that an elliptical galaxy is a collisionless system composed of several billion stars. Instead of following each star (or, in general, some other tracer, see below) one can operate in the six-dimensional phase-space volume $\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{v}$ around the position determined with $\mathbf{x}$ and velocity $\mathbf{v}$. Then, the distribution function[5] f is defined. This is obviously a non-negative function, i.e., $f \geq 0$. The quantity $f(\mathbf{x},\mathbf{v},t)\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{v}$ at time t determines the probability that a given star has phase-space coordinates in the given range. For simplicity, we may assume that all stars are identical and thus using the definition of the distribution function we have the following normalization:

$$\int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{v} f(\mathbf{x},\mathbf{v},t) = 1,$$

where the integral is taken over the entire phase-space.

A given star moves through the phase space and thus the probability of finding it at a given location is a function of time. Since the function f evolves, the probability must be conserved and this is given by the continuity relation which describes the conservation of fluid mass:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (\rho \dot{\mathbf{x}}) = 0,$$

where ρ and $\dot{\mathbf{x}}$ are the density and velocity of the fluid, respectively. If we now apply this approach for the conservation of probability, we have:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{w}} \cdot (f \dot{\mathbf{w}}) = 0,$$

where $\mathbf{w} = (\mathbf{x},\mathbf{v})$ are the Cartesian coordinates. Using Hamilton's equations to eliminate $\dot{\mathbf{w}} = (\dot{\mathbf{q}},\dot{\mathbf{p}})$, the collisionless Boltzmann equation is obtained:

$$\frac{\partial f}{\partial t} + \dot{\mathbf{q}} \cdot \frac{\partial f}{\partial \mathbf{q}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = 0,$$

where $\mathbf{q}$ is the generalized position and $\mathbf{p}$ is generalized momentum. The following equations hold:

$$\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}} \quad ; \quad \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{q}},$$

with H being the Hamiltonian, $H = \frac{1}{2}v^2 + \Phi(\mathbf{x},t)$, where Φ denotes the gravitational potential. Therefore, the collisionless Boltzmann equation can be expressed as:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0, \tag{1}$$

and it can be written for different coordinate systems (see BT08 for expressions in the cylindrical and spherical polar coordinates).

As noted in BT08, one should bear in mind that stars in a given galaxy are born and they die and thus their flow through the phase space should be described using a term which quantifies this fact. However, as BT08 show, in ellipticals and also, for example, for PNe in ellipticals, this effect is negligible and the collisionless Boltzmann equation without additional terms can be used.

When studying the problem of DM in ETGs it is important to make a link between theoretical expressions and observables. To achieve this we use the following integral which provides the probability per unit volume of finding a given star at the position $\mathbf{x}$, regardless of its velocity $\mathbf{v}$:

$$\nu(\mathbf{x}) \equiv \int \mathrm{d}^3\mathbf{v} f(\mathbf{x},\mathbf{v}).$$

Now, if we multiply this probability by the total number of stars in the population, N, the number density of stars in the real space is obtained:

$$n(\mathbf{x}) \equiv N\nu(\mathbf{x}).$$

In the case of stars the quantity ν can be obtained using the luminosity density $j(\mathbf{x}) = L\nu(\mathbf{x})$, where L is the luminosity of the stellar population (see details in Binney and Merrifield 1998, hereafter BM98). If we divide the distribution function by ν we obtain the probability distribution of stellar velocities at the position $\mathbf{x}$:

$$P_{\mathbf{x}}(\mathbf{v}) = \frac{f(\mathbf{x},\mathbf{v})}{\nu(\mathbf{x})},$$

[4]In modern language this corresponds to our Galaxy.

[5]The distribution function is called "the holy grail of dynamicists" in the review of galaxy masses by Courteau et al. (2014).

and in case of external galaxies, such as ETGs which we study here, this quantity can be determined through the line-of-sight velocity distribution (LOSVD, also called the velocity profile, VP) (see BM98 and Samurović 2007). There, individual stars cannot be resolved, and one has to deal with integrated stellar light of the given galaxy which represents the average of stellar properties of numerous unresolved stars that lie along each line of sight (hereafter, LOS): each star has a slightly different LOS velocity and, therefore, its spectral features will be shifted by a different amount $\Delta u = c\Delta\lambda/\lambda = v_{\rm LOS}$, where λ is the wavelength and $\Delta\lambda = (v_{\rm LOS}/c)\lambda$ (considering that $v_{\rm LOS} \ll c$) and the final measured galactic spectrum will be both shifted and broadened. LOSVD is a function $F(v_{\rm LOS})$ that defines the fraction of the stars that contribute to the spectrum which have LOS velocities between $v_{\rm LOS}$ and $v_{\rm LOS} + {\rm d}v_{\rm LOS}$, to be equal to $F(v_{\rm LOS}){\rm d}v_{\rm LOS}$. If one assumes that all stars have identical spectra $S(u)$ (where u is the spectral velocity in the spectrum of the given galaxy), then the intensity that is received from a star with LOS velocity $v_{\rm LOS}$ is given with $S(u - v_{\rm LOS})$. When the summation over all stars is performed the following function is obtained:

$$G(u) \propto \int {\rm d}v_{\rm LOS} F(v_{\rm LOS}) S(u - v_{\rm LOS}).$$

The expressions given above show that if we know ν, $\bar{\mathbf{v}}$ and $\sigma^{\mathbf{2}}$ at each point of the model, we can determine the observable quantities $v_{\rm LOS}$ and $\sigma^2_{\rm LOS}$ for that theoretical model. Therefore, this provides the necessary link between observations and theory. More precisely, in equilibrium stellar systems, such as ETGs, it is the Jeans equations that provide the relations between observed quantities and the gravitational field.

It has become a standard to model the LOSVD as truncated Gauss-Hermite ($F_{\rm TGH}$) series which consists of a Gaussian that is multiplied by a polynomial (van der Marel and Franx 1993, also Gerhard 1993):

$$F_{\rm TGH}(v_{\rm LOS}) = \Gamma \frac{\alpha_0(w)}{\sigma} \exp(-\frac{1}{2}w^2) \left[1 + \sum_{k=3}^{n} h_k H_k(w)\right]$$

where Γ represents the line strength, $w \equiv (v_{\rm LOS} - \bar{v})/\sigma$, $\alpha_0 \equiv \frac{1}{\sqrt{2\pi}} \exp(-w^2/2)$, where $\bar{v}$ and σ are free parameters and h_k are constant coefficients (see Fig. 2). The function $H_k(w)$ is a Gauss-Hermite function, that is a polynomial of the order k, except for $H_0 = 1$. In most works, the series is truncated at $k = 4$ (although higher values are also possible).

With the aforementioned definitions, the LOSVD can be calculated by varying the values of $\bar{v}$, σ, h_3 and h_4 until the convolution of the function $F_{\rm TGH}(v_{LOS})$ with a template stellar spectrum best reproduces the observed galaxy spectrum: the early works were based on a single star (usually K-type star, see van der Marel and Franx 1993 and e.g., Samurović 2007 for some examples) and more recently various stellar libraries have been used in order to reduce as much as possible the so-called template mismatch. The optimal fit is obtained using a non-linear least-squares fitting algorithm. The publicly available penalized Pixel-Fitting (pPXF) code[6] (Cappellari and Emsellem 2004, upgrade in Cappellari 2017) is frequently used for extraction the stellar or gas kinematics. It is assumed that the spectrum of a given galaxy can be represented by a linear combination of stellar spectra broadened by the velocity dispersion and shifted in the logarithmic space to some radial velocity where the non-linear least-squares fitting procedure provides the best-fitting result.

In Fig. 1 we show the central spectrum of an elliptical galaxy NGC 4473 (upper part) together with a template spectrum of a K3 I star (lower part) from which one can see that the galactic spectrum is both broadened and redshifted (see the caption for details).

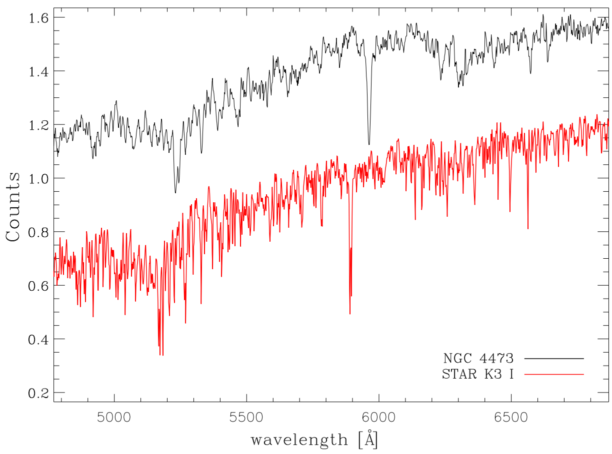

Fig. 1. *Upper part: the spectrum of the ETG NGC 4473 obtained using the MUSE integral-field spectrograph. Lower part: the spectrum of the best-fitting template star obtained using the Indo-U.S. Coudé Feed Spectral Library which contains 1273 stars covering the region from 3460 to 9464 Å at spectral resolution 1.35 Å(FWHM),* $\sigma \sim 30$ km s^{-1}, *R∼4200, see Valdes et al. (2004). The resolution of the star is reduced to the resolution of the MUSE spectrum i.e., 2.35 Å (FWHM) which corresponds to the instrumental resolution of* ~ 80 km s^{-1}. *The values of the extracted parameters are:* $v_{\rm GH} = 2243.8$ km s^{-1}, $\sigma_{\rm GH} = 192.5$ km s^{-1}, $h_3 = -0.014$ *and* $h_4 = 0.037$.

If the form of the LOSVD is close to the Gaussian form, then $\bar{v}$ and σ will be approximately equal to $\bar{v}_{\rm LOS}$ and $\sigma_{\rm LOS}$. Parameters h_3 and h_4 measure asymmetric and symmetric departures from the Gaussian respectively: the positive (negative) value of the h_3 parameter means that the distribution is skewed towards higher (lower) velocities with respect

[6] http://www-astro.physics.ox.ac.uk/∼mxc/software/#ppxf

to the systemic velocity. As for symmetric departures from the Gaussian, the detection of $h_4 > 0$ means that the distribution is more peaked than the Gaussian at small velocities with more extended high-velocity tails and for $h_4 < 0$ the distribution is more flat-topped than the Gaussian. The positive value of the h_4 parameter means that radial orbits dominate, whereas when $h_4 < 0$ the tangential orbits are dominant. The study of DM in ETGs is particulary sensitive with respect to the value of h_4 since this parameter plays a crucial role because it is constraining the level of tangential anisotropy: it is well known that the excess of tangential motions can mimic the existence of DM halos in these galaxies (Gerhard 1993).

The shape of $F_{\rm TGH}(v_{\rm LOS})$ as a function of h_3 and h_4 parameters is given in Fig. 2. Two largest samples of ETGs of comparable sizes for which the h_4 parameters were measured are given in Vudragović et al. (2016) who presented 84 ellipticals and 183 lenticulars galaxies (only their innermost regions are analyzed because the SDSS (Sloan Digital Sky Survey) spectra used provided only the innermost parts of ETGs) and in Cappellari et al. (2011) where 260 nearby ETGs were analyzed (see below). In Vudragović et al. (2016) the sample of 2180 galaxies (267 ETGs and 1788 spirals taken from the SDSS DR7 database) it was shown that we have a hint of an increase of the h_4 parameter from spirals to ETGs which indicates the increase of radial anisotropies; on the other hand, it was found that the h_3 parameter remains approximately constant over all morphological types. Another conclusion that was reached is

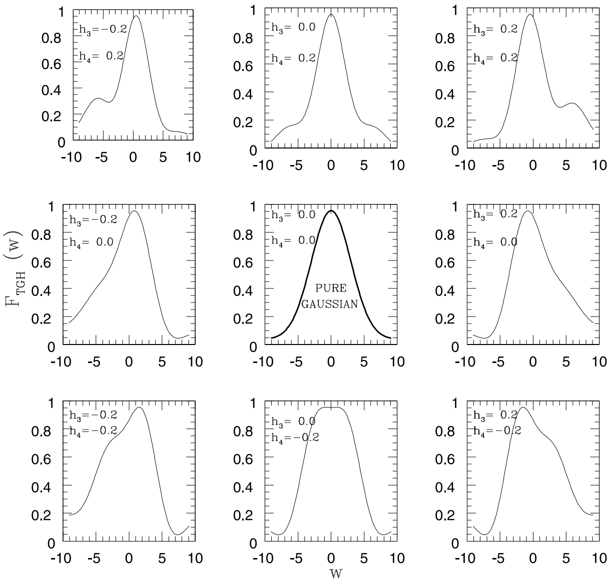

Fig. 2. *Several examples of the shape of the function $F_{\rm TGH}$. The parameters h_3 and h_4 are varied and the pure Gaussian function (the case with both h_3 and h_4 equal to zero) is shown in the center. Units of the variable w are arbitrary.*

related to the usage of established anisotropies: corrections[7] of the velocity dispersion associated with the h_4 parameter (see van der Marel and Franx 1993) were found to be meaningful and it was concluded that they should be applied whenever the signal-to-noise of the measured spectra reaches ~ 50.

Before writing down the final expressions for the Jeans equations, it is necessary to define the integrals of motion in a stationary potential Φ. The function $I(\mathbf{x}, \mathbf{v})$ is an integral of motion if and only if:

$$\frac{\mathrm{d}}{\mathrm{d}t} I[\mathbf{x}(t), \mathbf{v}(t)] = 0$$

along any orbit. This can be written as:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\partial I}{\partial \mathbf{x}} \cdot \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} + \frac{\partial I}{\partial \mathbf{v}} \cdot \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 0,$$

or

$$\mathbf{v} \cdot \frac{\partial I}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial I}{\partial \mathbf{v}} = 0.$$

Comparing the last equation with Eq. (1) one can see that the condition that I to be an integral of motion is equivalent to the condition that I be a steady-state solution of the collisionless Boltzmann equation.

For simplicity, in the discussion below we will limit ourselves to the simplest stellar systems, i.e., to the spherical stellar systems which is good approximation for at least some elliptical galaxies (of morphological type E0).

It is also important to define here the so called anisotropy parameter β:

$$\beta \equiv 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2} = 1 - \frac{\overline{v_\theta^2} + \overline{v_\phi^2}}{2\overline{v_r^2}},$$

which is equivalent to the h_4 parameter defined above (see Gerhard 1993) and describes the degree of anisotropy of the given system. If all orbits are circular, $\sigma_r = 0$ which means that $\beta = -\infty$ and if all orbits are radial, $\sigma_\theta = \sigma_\phi = 0$ which means that $\beta = 1$. Therefore, as in the case of the h_4 parameter, the distribution functions with $\beta > 0$ are *radially* biased and those with $\beta < 0$ are *tangentially* biased. The case for which $\beta = 0$ is the isotropic case. We note that for a galaxy in which both the density and the velocity structures are invariant under rotations about the galactic center, i.e., for a galaxy that does not rotate, the following equation holds: $\overline{v_\theta^2} = \overline{v_\phi^2}$. This means that the β-parameter becomes:

$$\beta \equiv 1 - \frac{\overline{v_\theta^2}}{\overline{v_r^2}},$$

see e.g., Binney and Tremaine (1987). Below, we will discuss the role anisotropies play in ETGs and their total dynamical mass.

We now proceed with an integration of Eq. (1):

$$\int d^3\mathbf{v} \frac{\partial f}{\partial t} + \int d^3\mathbf{v} v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \int d^3\mathbf{v} \frac{\partial f}{\partial v_i} = 0,$$

[7] $\sigma_{\rm corr} = \sigma_{\rm GH}(1 + \sqrt{6}(h_4)_{\rm GH})$, where the index "GH" is related to estimates coming from the Gauss-Hermite fits.

and taking into account that at sufficiently large $\mathbf{v}$ we have $f(\mathbf{x}, \mathbf{v}, t) = 0$ which means that there are no stars moving infinitely fast. Using the expression derived for ν we obtain:

$$\frac{\partial \nu}{\partial t} + \frac{\partial \left(\nu \overline{v}_i \right)}{\partial x_i} = 0, \tag{2}$$

which is an equation of continuity of probability. Following the steps given in BT08 omitted here for brevity we arrive to an equation analog to Euler's equation of fluid flow:

$$\nu \frac{\partial \overline{v}_j}{\partial t} + \nu \overline{v}_i \frac{\partial \overline{v}_j}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \frac{\partial \left(\nu \sigma_{ij}^2 \right)}{\partial x_i}. \tag{3}$$

Eqs. (2) and (3) are the Jeans equations.

BT08 discusses the cases when the closure of the equations is possible and the reader is referred to the details provided there. For our purposes it is now important to write the Jeans equations in spherical coordinates. The collisionless Boltzmann equation (see above and BT08) is written using the following expressions for momenta:

$$p_r = \dot{r} = v_r \quad ; \quad p_\theta = r^2 \dot{\theta} = r v_\theta$$

and:

$$p_\phi = r^2 \sin^2 \theta \dot{\phi} = r \sin \theta v_\phi .$$

Assuming that the system is spherical and time independent one obtains (see BT08 for details):

$$\frac{\mathrm{d} \left(\nu \overline{v_r^2} \right)}{\mathrm{d} r} + \nu \left(\frac{\mathrm{d} \Phi}{\mathrm{d} r} + \frac{2\overline{v_r^2} - \overline{v_\theta^2} - \overline{v_\phi^2}}{r} \right) = 0.$$

Using the β parameter defined above, the following equation is obtained:

$$\frac{\mathrm{d} \left(\nu \overline{v_r^2} \right)}{\mathrm{d} r} + 2 \frac{\beta}{r} \nu \overline{v_r^2} = -\nu \frac{\mathrm{d} \Phi}{\mathrm{d} r}$$

and we use it to write down the form of the Jeans equation used in our analysis below. One of the earliest application of this formula was in Duncan and Wheeler (1980) where the elliptical galaxy M87 was studied.

This is the form of the spherical Jeans equation most frequently used in the analysis of DM in this paper:

$$\frac{\mathrm{d} \sigma_r^2}{\mathrm{d} r} + \sigma_r^2 \frac{(2\beta + \alpha)}{r} = a_{\mathrm{N;M}} + \frac{v_{\mathrm{rot}}^2}{r}, \tag{4}$$

and is applied in both approaches, the Newtonian ("N") and MOND ("M") where $a_{\mathrm{N;M}}$ is an acceleration term calculated separately for each methodology. In the Newtonian approach $a_{\mathrm{N}} = -\frac{GM(r)}{r^2}$ and for the MOND models $a_{\mathrm{M}} \mu(a_{\mathrm{M}}/a_0) = a_{\mathrm{N}}$. The α parameter defined as $\alpha = \mathrm{d} \ln \nu / \mathrm{d} \ln r$ is the slope of tracer density ν. The Newtonian acceleration is expressed as $a_{\mathrm{N}} = a \mu(a/a_0)$, where a is the MOND acceleration and a_0 is the universal constant ($a_0 = 1.35^{+0.28}_{-0.42} \times 10^{-8}$ cm s^{-2}, Famaey et al. 2007). The rotational component is always taken into account in models when $v_{\mathrm{rot}} > 0$ km s^{-1} through the expression[8]:

$$v_{\mathrm{rms}} = \sqrt{v_{\mathrm{rot}}^2 + \sigma^2}. \tag{5}$$

Dynamical models based on the Jeans equation suffer from a well-known problem of mass-anisotropy degeneracy between the assumed mass density and the velocity distribution of the stellar system (Binney and Mamon 1982, Tonry 1983). This may produce an erroneous mass estimate of the given galaxy which is especially important in the analysis of the DM content. In the literature several approaches can be found to lift this degeneracy: see, for example, the procedures of fitting the LOS velocity kurtosis profiles together with the LOS velocity dispersion profiles (Łokas 2003, Łokas and Mamon 2003, Richardson and Fairbairn 2013) and the method that reconstructs the unique kinematic profile for some assumed free functional form of the mass density based on flexible B-spline functions for representation of the radial velocity dispersion (Diakogiannis et al. 2014). Wolf et al. (2010) derived an accurate mass estimator for dispersion-supported stellar systems such as ETGs. They relied on the spherical Jeans equation and showed how the mass ($M_{1/2}$) enclosed within the 3D deprojected half-light radius ($r_{1/2}$) can be determined by using a simple formula, assuming that the projected velocity dispersion profile is fairly flat near the effective radius R_{e}:

$$M_{1/2} = 3G^{-1} \left\langle \sigma_{\mathrm{LOS}}^2 \right\rangle r_{1/2} \simeq 4G^{-1} \left\langle \sigma_{\mathrm{LOS}}^2 \right\rangle R_{\mathrm{e}}. \tag{6}$$

When working within the MOND framework, three MOND interpolating functions are most frequently used in the analysis of ETGs (but see also below). A simple MOND formula (Famaey and Binney 2005) is given by (the expression $x \equiv a/a_0$ is valid in all MOND cases):

$$\mu(x) = \frac{x}{1 + x}. \tag{7}$$

A standard MOND formula (Sanders and McGaugh 2002) is:

$$\mu(x) = \frac{x}{\sqrt{1 + x^2}}. \tag{8}$$

The toy MOND model (Bekenstein 2004) is given as:

$$\mu(x) = \frac{-1 + \sqrt{1 + 4x}}{1 + \sqrt{1 + 4x}}. \tag{9}$$

[8]See Figure 3 below where the velocity dispersion profile of NGC 4473 was modeled.

The expressions for the circular velocities for all three functions are given in Samurović and Ćirković (2008a).

The projected line-of-sight velocity dispersion is calculated in the following manner (Binney and Mamon 1982):

$$\sigma_p^2(R) = \frac{\int_R^{r_t} \sigma_r^2(r) \left[1 - (R/r)^2\beta\right] \rho(r)(r^2 - R^2)^{-1/2} r dr}{\int_R^{r_t} \rho(r)(r^2 - R^2)^{-1/2} r dr}, \tag{10}$$

where the truncation radius $r_{\rm t}$ extends well beyond the outermost observed kinematical point of the given galaxy.

3. INTEGRATED STELLAR SPECTRA

3.1. Long-split spectra

The study of DM in ETGs based on the stellar kinematics started with investigation of the integrated stellar spectra using the Jeans equation together with the analysis of mass profiles based on X-ray observations. For the reasons of completeness other methods of detecting DM will also be mentioned below (such as Schwarzschild, distribution functions and quadratic programming, QP, methods). In the early work of Binney et al. (1990) the two-integral Jeans modeling procedure was introduced. They analyzed long-slit spectra of three ETGs (NGC 720, NGC 1052 and NGC 4697) and modeled the observed velocity dispersion profiles. They relied on photometry to construct axisymmetric mass models of the three ETGs and combined them with the Jeans equations to model the velocity dispersions. Although they did not explicitly deal with the DM problem, they posed the fundamental question to be addressed in ETGs: "Is the mass-to-light ratio constant?" The spectroscopic data which they had extended out approximately to at most one effective radius. They found that the observed data are consistent with the constant mass-to-light ratio of all three ETGs in these inner regions studied. The same technique was then used in van der Marel et al. (1990) on other four ETGs (NGC 3379, NGC 4261, NGC 4278 and NGC 4472), to distances up to $\sim 1R_{\rm e}$, who found that constant mass-to-light ratios can provide successful fits to the observed velocity dispersion profiles. Cinzano and van der Marel (1994) studied the elliptical galaxy NGC 2974 (classified as E4) and in their models they included both h_3 and h_4 parameters besides the velocity and velocity dispersion. They again used both the long-slit spectra and photometric data and concluded by comparing their data with HI observations that this galaxy possesses a dark halo with mass greater than that of the luminous component (interior to $\sim 3.6R_{\rm e}$). Their spectroscopic observations extended out to $\sim 0.5R_{\rm e}$ and they found no evidence for DM there. The spectroscopic study of ETGs beyond $1R_{\rm e}$ was done in Carollo et al. (1995): they studied one E1 galaxy (NGC 2434) and three E3 galaxies (NGC 2663, NGC 3706 and NGC 5018) and using the Jeans equations they found that all the galaxies except NGC 2663 must possess a dark halo.

The research of DM in ETGs also used some other theoretical approaches to infer the possible contribution of DM in their mass content. For example, Bertin et al. (1992) used two-component, spherically symmetric, collisionless, self-consistent models to study the contribution of DM in ellipticals. They also relied on the Jeans equation for the luminous component and applied their approach in Saglia et al. (1992). They fitted their models to the observed photometric and kinematic profiles of several galaxies. They reached the conclusion that the amount of DM within $1R_{\rm e}$ is not too large: in the case of NGC 4472 they found that DM must be present.

Rix et al. (1997) used Schwarzschild's (1979) method for construction of axisymmetric and triaxial models of galaxies in equilibrium without explicit knowledge of the integrals of motion. In their detailed dynamical modeling they relied on velocity, velocity dispersion and Gauss-Hermite parameters h_3 and h_4 in the case of the galaxy NGC 2434 (from Carollo et al. 1995). They concluded that this galaxy contains a lot of DM and claimed that about half of the mass within $1R_{\rm e}$ is dark. The post-merger elliptical galaxy NGC 1700 was studied in Statler et al. (1996) who analyzed its stellar kinematical fields out to $\sim 4R_{\rm e}$. In a subsequent paper Statler et al. (1999) found, using the minor modification of the Binney et al. (1990) of solving the Jeans equations (and also three-integral QP models), that NGC 1700 must have a radially increasing mass-to-light ratio. They concluded that NGC 1700 "appears to represent the strongest stellar dynamical evidence to date for dark halos in elliptical galaxies". Saglia et al. (2000) modeled the central galaxy of the Fornax cluster NGC 1399 out to $\sim 2.5\ R_{\rm e}$ using the method based on distribution functions and marginally detected the influence of the dark component that starts from 1.5 $R_{\rm e}$. This same galaxy was the subject of work of Samurović and Danziger (2006) who analyzed it using both X-rays and GCs and detected the influence of DM beyond $\sim 3R_{\rm e}$.

De Bruyne et al. (2001) modeled two ETGs, NGC 4649 (M60) and NGC 7097, using a three-integral QP method and found that in the case of NGC 4649 a constant mass-to-light ratio ($M/L_V = 9.5$) fit can provide a good agreement with the data and that a marginally better fit can be obtained including 10% of DM at 1.2 $R_{\rm e}$. In the case of NGC 7097 both kinematic and photometric data can be fitted out to 1.6 $R_{\rm e}$ using a constant mass-to-light ratio $M/L_V \sim 7.2$. Cretton et al. (2000) modeled the giant elliptical galaxy NGC 2320 using the Schwarzschild orbit superposition method and found that models with radially constant mass-to-light ratio and logarithmic models with DM provide comparably good fits to the data and have a similar dynamical structure (but note that the mass-to-light ratio in the V-band is rather large: ~ 15 for the mass-follows-light models and ~ 17 for the logarithmic models).

Some early attempts of the Schwarzschild modeling of central parts of ETGs include the works by van der Marel et al. (1998) in which M32 was analyzed, the Cretton and van der Bosch (1999) paper in which axisymmetric models of NGC 4342 were analyzed, the Gebhardt et al. (2000) paper in which a black hole in the center of NGC 3379 was modeled, the Gebhardt et al. (2003) paper with the sample of 12 ellipticals that were analyzed using the axisymmetric approach. Finally, we mention the paper by Cappellari et al. (2002) who, using the Schwarzschild formalism, modeled in detail, the internal parts of one ETG (IC 1459) previously analyzed to several effective radii using long-slit spectra (see also below).

Gerhard et al. (1998) measured the LOS velocity profiles of the round elliptical (E0 type) NGC 6703 out to $\sim 2.6R_\mathrm{e}$ in an attempt to constrain the DM halo of this object. They obtained long-slit spectra of this galaxy and extracted full LOSVD which they modeled assuming spherical distribution functions to constrain the mass distribution and the anisotropy of the stellar orbits in this elliptical: they concluded that DM is needed in the outer part of the galaxy to successfully fit the observed data. Kronawitter et al. (2000) modeled a sample of 21 ellipticals out to 1-2 R_e using the technique of Gerhard et al. (1998) and Saglia et al. (2000): for three of them (NGC 2434, NGC 7507, NGC 7626) they found that models based on luminous matter should be ruled out.

Samurović and Danziger (2005) studied four ETGs using long-slit spectra which extend to $\sim 1-3R_\mathrm{e}$. The galaxies IC 1459, IC 3370, NGC 3379 and NGC 4105 were analyzed using two-integral Jeans models taking into account the full LOSVD and also making a comparison with X-ray observations, where possible. The Jeans dynamical modeling could not unambiguously detect the significant amounts of DM in four ETGs studied within the uncertainty of the modeling. For example, the results related to IC 1459 suggested that for this galaxy with a counter-rotating core the h_4-parameter in its outer parts has large values thus implying the existence of radial anisotropies; also, it was found that DM in this galaxy is not dominant in the interior up to $\sim 3R_\mathrm{e}$.

In Salinas et al. (2012) the kinematics of the isolated elliptical NGC 7507 was studied, and major and minor axis long-slit spectra were obtained in order to establish the existence of a DM halo there. The authors used the spherical Jeans equation and found that the models without DM provide an excellent fit of the observed data. On the other hand, interestingly, they found that the MOND predictions overestimate the observed velocity dispersion. Therefore, they concluded that although DM may be present it should be in conjunction with a strong radial anisotropy for which they discovered some indications.

3.2. Integral field spectroscopy

The integral field spectroscopy (IFS) is a new useful technique which provides an improvement in the study of integrated stellar spectra over the long-slit observations because of larger spatial coverage. In Murphy et al. (2011) the integral field spectrograph VIRUS-P was used and two-dimensional stellar kinematics of M87 out to 238 arcsecs was presented: using a large set of axisymmetric, orbit-based dynamical models the authors found clear evidence for a massive DM halo in M87, which is in agreement with early results based on GCs mentioned above. Three years later, in Raskutti et al. (2014) the same instrument with a large 107×107 arcsec^2 field of view, now renamed as the Mitchell Spectrograph on the 2.7 m Harlan J. Smith telescope at McDonald Observatory, was used in a study of a two-dimensional kinematic LOSVD analysis out to $\sim 2-5R_\mathrm{e}$ of 33 massive ellipticals with stellar velocity dispersions above 150 km s^{-1}.

Here, we list some of the most important results obtained by several observing projects using this technique related to the subject of the present paper (see the review in Cappellari 2016 where other applications are described as well).

The first generation of IFS surveys important for the study of ETGs included the following projects: SAURON[9] (Spectroscopic Areal Unit for Research on Optical Nebulae) (Bacon et al. 2001), ATLAS$^{\mathrm{3D}}$[10] (Cappellari et al. 2011) and CALIFA[11] (Calar Alto Legacy Integral Field Area Survey, Sánchez et al. 2012) which all completed their tasks. Below some of their most important achievements are listed.

SAURON: In Emsellem et al. (2004) the maps of integral-field absorption kinematics of 48 ETGs which include velocity, velocity dispersion and the Gauss-Hermite parameters h_3 and h_4 out to about one effective radius were presented. In Cappellari et al. (2006) the analysis of correlations between the dynamical mass-to-light ratio and other global observables of ETGs were performed: the authors used the two-integral Jeans and three-integral Schwarzschild dynamical models for a sample of 25 ETGs to approximately one effective radius. They concluded that DM contributes to ~ 30 per cent of the total dynamical mass and they speculate that fast-rotating galaxies have lower DM fractions than the slow-rotating (more massive) galaxies.

ATLAS$^{\mathrm{3D}}$: In Cappellari et al. (2011) the complete sample of 260 nearby ETGs within 42 Mpc was analyzed and the authors used the Gauss-Hermite parametrization to extract h_3 and h_4 parameters relying on stellar template as an optimal linear combination of stars from the MILES library (Sánchez-Blázquez et al. 2006). Each template was

[9] http://www.strw.leidenuniv.nl/sauron

[10] http://www-astro.physics.ox.ac.uk/atlas3d

[11] http://califa.caha.es

separately determined for every galaxy in their sample. In a subsequent paper (Cappellari et al. 2013) the ATLAS^{3D} team analyzed the problem of the mass-to-light ratio and DM in their sample. They made the Jeans axisymmetric dynamical models taking into account orbital anisotropies and they included a DM halo in their calculation. The authors relied on the JAM (Jeans Anisotropic Multi-Gaussian Expansion) code developed by Cappellari (2008)[12]. They also calculated total mass-to-light ratios and DM fractions (median value of $f_{\rm DM}$ = 13 per cent) within approximately one effective radius. Chae et al. (2018) recently analyzed spherical Jeans models of 24 nearly round pure-bulge galaxies from the ATLAS^{3D} sample together with 4201 SDSS galaxies and found, among other things, an indication that the stellar mass-to-light ratio gradient has a positive correlation with stellar mass.

CALIFA: The CALIFA sample of nearby galaxies includes approximately 600 objects of which approximately 200 galaxies belong to the ETG class (Sánchez et al. 2012). In most cases they cover the data out to R_{25} radius (the isophotal radius where surface brightnes μ_B reaches 25 mag/arcsec^2). In Falcón-Barroso et al. (2017) the stellar kinematic maps (which include velocities and velocity dispersions) of approximately 300 galaxies are presented featuring various morphologies across the Hubble sequence, from ellipticals to late-type spirals up to several effective radii. In Zhu et al. (2018) the problem of DM in their sample was analyzed using the Schwarzschild approach. After a detailed analysis they conclude that: "Although our dynamical models yield precise measurements of the total enclosed mass, the separation into stellar, gas and dark matter masses is much less certain, owing to degeneracies and unavailability of cold gas measurements."

The second generation of IFS active in research of ETGs[13] includes the following surveys: MUSE[14] (Multi Unit Spectroscopic Explorer) (Bacon et al. 2010), MASSIVE[15] (Ma et al. 2014), SAMI[16] (The Sydney AAO Multi Integral Feld) (Bryant et al. 2015), and MaNGA[17] (Mapping Nearby Galaxies at APO) (Bundy et al. 2015).

MUSE: MUSE is a second generation instrument installed on the Nasmyth focus of UT4 at the Very Large Telescope (VLT) of the European Southern Observatory (ESO). Several ETGs were explored with MUSE and we mention here the case of the low-luminosity S0 galaxy NGC 5102 (Mitzkus et al. 2017) who used the JAM modeling of this galaxy to investigate the initial mass function (IMF) and the DM halo of this object. Their data covered the inner region of the galaxy, the interior to $\sim 1R_{\rm e}$. In their models they included DM to reproduce the observed kinematics. The model with an NFW (Navarro et al. 1997) DM halo required a DM fraction of 0.37 ± 0.04 inside one effective radius and a heavy IMF.

MASSIVE: The MASSIVE Survey is a volume-limited, multi-wavelength, IFS and photometric survey of the structure and dynamics of approximately 100 most massive ETGs within a distance of 108 Mpc in diverse galaxy environments. In Veale et al. (2017) the authors analyzed the spatially resolved stellar angular momentum, velocity dispersion, and higher moments of the 41 most massive local ETGs in their sample. They analyzed their volume-limited sample of the highest end of the galaxy mass function and calculated the kinematic parameters out to $1R_{\rm e} - 4R_{\rm e}$ of each galaxy: they measured the 2D spatial distributions of six Gauss-Hermite moments of the LOSVD (velocity, velocity dispersion, $h_3 - h_6$ parameters). Among other things, they discovered that the luminosity-weighted average h_4 parameter, $< h_4 >_{\rm e}$, is positive for all 41 ETGs in their sample. This is in agreement with the study of Vudragović et al. (2016) who found that radial anisotropies with $h_4 > 0$ dominate in the innermost parts of ETGs (and especially ellipticals). In the subsequent paper Veale et al. (2018) studied the stellar velocity dispersion profiles and environmental dependence of 90 ETGs. They discovered a clear positive correlation between the gradients of the outer velocity dispersion profile and the gradients of the Gauss-Hermite h_4 parameter within one effective radius. The X-ray properties of 33 ETGs from the MASSIVE survey were presented in Goulding et al. (2016).

SAMI: The SAMI Galaxy Survey has the intention of creating a large survey of 3000 galaxies across a large range of environment. In van de Sande (2017) the stellar kinematics on a sample of 315 galaxies without a morphological selection was measured. They used the pPXF code to extract full LOSVD from the integral field spectra which included velocity, velocity dispersion and h_3 and h_4 parameters. For most of their galaxies the kinematic data extend between one and two effective radii. In Scott et al. (2018) the second major release of data from the SAMI Galaxy Survey was presented: data for 1559 galaxies (about 50 per cent of the full survey) were provided. The data which extend out to $\sim 1R_{\rm e}$ include stellar kinematic and stellar population value-added products derived from absorption line measurements. For approximately 60 percent of

[12] `http://www-astro.physics.ox.ac.uk/~mxc/software/#jam`, see also Cappellari (2015)

[13] We note that the VENGA (VIRUS-P Exploration of Nearby Galaxies) IFS project (Blanc et al. 2013) maps the disks of 30 nearby spiral galaxies and therefore will not be described here.

[14] `http://muse-vlt.eu/science`

[15] `http://blackhole.berkeley.edu`

[16] `http://sami-survey.org`

[17] `https://www.sdss.org/surveys/manga`

the galaxies from their sample the authors measured $(v/\sigma)_{\rm e}$ and the spin parameter proxy $(\lambda_R)_{\rm e}$ at the position of one effective radius.

MaNGA: The MaNGA survey is a part of the SDSS-IV, the latest generation of the SDSS which for the first time uses the Sloan spectrographs to make spatially resolved maps of individual galaxies. The MaNGA project provides spectral measurements across the face of each of approximately 10000 nearby galaxies using 17 simultaneous "integral field units" (IFUs), each composed of arrays of optical fibers. The goal of the MaNGA project is to provide two-dimensional maps of stellar velocity and velocity dispersion, mean stellar age and star formation history, stellar metallicity, element abundance ratio, stellar mass surface density, ionized gas velocity, ionized gas metallicity, star formation rate and dust extinction for their large sample of galaxies chosen to span a stellar mass interval of nearly three orders of magnitude. One of the aims of the MaNGA survey is to create, using the same instrument, a stellar spectral library (named MaStar) with a comprehensive stellar parameter coverage, a large sample size, and high quality calibrations, see Yan et al. (2018). This will be important for modeling of the stellar populations and thus inferring the DM content in ETGs. The MaNGA database (MPL5 dataset) was for example used in the work by Rong et al. (2018) who studied the recently derived radial acceleration relation (RAR)[18] in ETGs. They used the Jeans dynamical modeling and found that there exists a 1σ significant difference between the acceleration relations of the fast and slow rotators and propose that the acceleration relation deviation of slow rotators may be due to more galaxy merger events, which would disrupt the original spins and correlated distributions of baryons and DM orbits in galaxies. The question of the RAR relation will also be addressed below in the context of study of GCs in massive ETGs. Durazo et al. (2018) also used the MaNGA database to study approximately 300 extremely low rotating ellipticals and showed that dynamics of this type of systems is consistent with expectations coming from MOND, for central velocity dispersion between 60 km s^{-1} and 280 km s^{-1} and asymptotic velocity dispersion between 28 km s^{-1} and 250 km s^{-1} and that a universal velocity dispersion profile accurately describes such pressure supported systems. It is important to note that the MaNGA survey aimed to provide a uniform radial coverage to $1.5R_{\rm e}$ for two-thirds and $2.5R_{\rm e}$ for one-third of the final sample (Bundy et al. 2019), so the interior regions up to $\sim 3R_{\rm e}$ are covered.

To summarize this part dedicated to the study of the integrated stellar spectra: the second generation of IFS is still producing results and one may expect that the study of ETGs and DM in them will certainly benefit from future papers by the aforementioned projects and works based on their databases.

4. VARIOUS TRACERS OF GRAVITATIONAL POTENTIAL

If we want to estimate the mass of ETGs beyond several effective radii $(2R_{\rm e} - 3R_{\rm e})$ other methods and approaches are needed. One may use the X-ray approach which does not suffer from the mass-anisotropy degeneracy (see e.g., Samurović 2007 and references therein) but the existence of hydrostatic equilibrium upon which the theory is based is sometimes uncertain (see Diehl and Statler 2007). Also, not all ETGs posses an X-ray halo (see the example of the massive elliptical galaxy IC 3370 in Samurović and Danziger 2005). Another interesting approach which has been extensively used in the last decade is that of using tracers such as PNe and GCs in ETGs. To analyze the existence of DM in these galaxies once again it is necessary to solve the Jeans equation (Eq. (4)) and several steps are necessary in such an endeavor. First, one needs a list of the observed objects (PNe, or GCs) with their radial velocities and galactocentric distances. Then it is necessary to plot the surface density profile of the given galaxy to calculate $\alpha = {\rm dln}\nu/{\rm dln}r$ which determines the slope of tracer (PNe, or GCs) density ν (see, for example, Figs. 6 and 7 in Samurović 2014 where profiles of 10 ETGs were plotted). One also needs to estimate the β parameter which determines orbital anisotropies in the motion of the observed tracers. It must be noted here that, with current observational situation, one cannot hope to measure accurately these anisotropies throughout the entire given galaxy: one may expect to establish roughly whether anisotropies do exist and their type (i.e., radial or tangential) but to have a precise estimate of the anisotropies one would need at least 500 objects per bin (see Merritt 1997) which is not available at the moment even for ETGs with large number of tracers.

Using the observed data and the Jeans equation one fits the measured values of the velocity dispersion. The value of the velocity dispersion is calculated as:

$$\sigma_{\rm LOS,\ obs} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(v_i - v_{\rm sys})^2}. \qquad (11)$$

The error of the velocity dispersion is given as:

$$\Delta\sigma_{\rm LOS,\ obs} = \frac{\sigma_{\rm LOS,\ obs}}{\sqrt{2(N-1)}}. \qquad (12)$$

The departures from the Gaussian with their errors are calculated using the standard formulas for unbiased estimators (Joanes and Gill 1998). The asymmetric departure is given by:

[18]This empirical relation links the observed acceleration a with the acceleration predicted by the Newtonian dynamics based on the distribution of the observable matter $a_{\rm N}$ (McGaugh et al. 2016), see also below. This relation was devised using 175 spiral galaxies coming from the Spitzer Photometry and Accurate Rotation Curves (SPARC) database.

$$s_3 = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} \left(\frac{v_i - v_{\rm sys}}{\sigma} \right)^3 \pm \sqrt{\frac{6}{N}}. \tag{13}$$

The symmetric departure (corresponds to the h_4 or β parameter) is:

$$s_4 = \frac{N(N+1)}{(N-1)(N-2)(N-3)} \sum_{i=1}^{N} \left(\frac{v_i - v_{\rm sys}}{\sigma} \right)^4 - 3\frac{(N-1)^2}{(N-2)(N-3)} \pm \sqrt{\frac{24}{N}}, \tag{14}$$

where: N is the number of tracers in the bin, v_i is the radial velocity of the i-th tracer and $v_{\rm sys}$ the average line-of-sight velocity of all given tracers in the given galaxy.

4.1. Planetary nebulae (PNe)

First, we will briefly describe the usage of PNe in investigation of the DM problem in ETGs. PNe are very useful tool for this purpose since they are detectable even in moderately distant galaxies through their strong emission lines.

One of the most studied ETGs using both PNe and GCs is a giant elliptical NGC 5128 (Centaurus A).[19] An early attempt of usage of PNe as tracers of the total dynamical mass was in Hui (1993) who applied the spherical Jeans equation to model the observed velocity dispersion profile. Over 400 PNe were identified and assuming the Hernquist (1990) mass model, an increase of the total mass-to-light ratio was deduced (in the B-band, from 3.5 in the inner region to 10 at 25 kpc) which implied the existence of a DM halo in the galaxy. Hui et al. (1995) again used PNe as tracers to confirm this previous result: they found that the mass-to-light ratio in its central region of approximately 3.9 rises to approximately 10 at $5R_{\rm e}$ (in the B-band), thus again indicating the existence of dark halo. Later, again relying on PNe using imaging and spectroscopy Peng et al. (2004) detected 780 spectroscopically confirmed PNe. They discovered that PNe exist at distances out to 80 kpc ($\sim 15R_{\rm e}$) and found that DM is necessary to explain the observed stellar kinematics. However, their calculated value of the mass-to-light ratio is *much* lower than that expected from determinations based on the usage of the X-ray halos: using the spherical Jeans equation they found the total dynamical mass $\sim 5\times10^{11}M_\odot$ with $M/L_B \sim 13$ within 80 kpc. Bahcall et al. (1995) compiled the mass-to-light ratios from literature, and concluded that at 80 kpc one should expect $M/L_B \sim 112 \pm 28$. Hui et al. (1995) used the Jeans equation and fitted the velocity dispersion profile of NGC 5128; they measured the dynamical mass of $3.1 \times 10^{11}M_\odot$ within 25 kpc. This estimate is significantly lower than that measured by Forman et al. (1985) from ROSAT data who calculated a total mass of $1.2 \times 10^{12}M_\odot$ within 20 kpc. Also, van Gorkom et al. (1990) estimated the dynamical mass of NGC 5128 using HI synthesis observations and found that it is much lower than that obtained by using the X-ray halo: they found that the total mass is $2.5 \times 10^{11}M_\odot$, and the mass within $1.2R_{\rm e}$ is $1.2 \times 10^{11}M_\odot$. The ETG NGC 5128 was again the subject of the Jeans analysis based on its PNe in Samurović (2010) where it was found that the isotropic Newtonian mass-follows-light models without DM may provide successful fits out to $\sim 6.4R_{\rm e}$ and that to obtain a good fit in the outermost region ($\sim 10.7R_{\rm e}$) either DM or the existence of tangential anisotropies is needed; MOND models were also analyzed and it was discovered that no isotropic MOND model without DM can provide a successful fit interior to $\sim 6.4R_{\rm e}$ and that interior to $\sim 8R_{\rm e}$ only a standard model with tangential anisotropies can fit the observed velocity dispersion without the need of DM.

Another interesting example of usage of PNe methodology in DM research is that of the elliptical galaxy NGC 3379. Ciardullo et al. (1993) measured radial velocities of 29 PNe (out to 3.8 $R_{\rm e}$, assuming $1R_{\rm e} = 54.8$ arcsec from Capaccioli et al. 1990) and used the QP method to draw the conclusion that the mass-to-light ratio is $M/L_B \sim 7$ and that there is no need for DM. Ten years later, Romanowsky et al. (2003) observed PNe in three galaxies (NGC 821, NGC 3379 and NGC 4494) and relying on the spherical Jeans models they confirmed the lack of DM in NGC 3379 using a much larger sample of 109 PNe (out to ~ 3.5 $R_{\rm e}$, assuming the same value of the effective radius stated above). Samurović and Danziger (2005) analyzed the region of the NGC 3379 interior to $\sim 1R_{\rm e}$ with long-slit spectra (see above) using the Jeans equation and found that DM is not needed in the regions they studied; their analysis of X-rays there also suggested that interior to $\sim 1.5R_{\rm e}$, the cumulative mass-to-light ratio is in agreement with that based on the dynamical Jeans modeling. It was found that the result of Romanowsky et al. (2003) is in agreement with the MOND theory (see, for example, Milgrom and Sanders 2003). Tiret et al. (2007) analyzed again NGC 3379 using both Cold Dark Matter (CDM) models and MOND and solved the Jeans equation using several tracers of the gravitational potential (stars, PNe, and satellite galaxies). They found that only the MOND models provided good fits to the observational data whereas both the standard CDM model and the Newtonian purely baryonic models encountered problems on small and large scale, respectively.

Sluis and Williams (2006) used the Rutgers Fabry–Pérot[20] in order to search for PNe in NGC 3379 and three other ETGs: they detected 54

[19] This galaxy will be among the objects analyzed in the next subsection where some results based on both PNe and GCs as well as X-rays for NGC 5128 will be presented.

[20] The Rutgers Fabry-Peérot (RFP) is a narrowband (< 2Å) filter with a tunable central wavelength used for investigation of the redshifted [O III] $\lambda5007$ emission line used for measurement of radial velocities of PNe.

PNe in NGC 3379 and again using the spherical Jeans equation found that within $\sim 3R_{\rm e}$ the total mass-to-light ratio of this galaxy $M/L_B \sim 5$ implies a very low amount of DM in the studied region (interior to ~ 130 arcsec). Douglas et al. (2007) using Planetary Nebula Spectrograph (PN.S) observations of NGC 3379 detected 214 PNe of which 191 are ascribed to NGC 3379 and 23 to the companion galaxy NGC 3384. They produced a series of Jeans dynamical models and found that inside $5R_{\rm e}$ the mass-to-light ratio in the B-band is equal to $8-12$ and that the DM fraction inside this radius is below 40 percent. de Lorenzi et al. (2009) used the PN.S data together with combined photometry, long-slit spectroscopic data, SAURON absorption line kinematics to create dynamical models with DM. For the data that extend out to $\sim 7R_{\rm e}$ using the aforementioned NMAGIC code they found that they could only weakly exclude models without DM. They concluded that assuming spherical symmetry, the observational data are consistent i) with near-isotropic systems dominated by the stellar mass out to the last kinematic data points, and ii) with models of moderately massive halos for which the outer parts are strongly radially anisotropic, i.e., $\beta \gtrsim 0.8$ at $7R_{\rm e}$. For the radially anisotropic models, the visible stellar component dominates in the centre and the DM fraction becomes equal to ~ 60 percent of the total dynamical mass at $7R_{\rm e}$.

PN.S was again used in Napolitano et al. (2009) in the analysis of DM in the elliptical galaxy NGC 4494. The authors measured the positions and velocities of 255 PNe out to $\sim 7R_{\rm e}$ (25 kpc) and also presented a wide-field surface photometry and long-slit stellar kinematics to find that spatial and kinematical distributions of the PNe agree with the field stars in the region where different observations overlap. They detected the declining velocity dispersion profile and used spherical Jeans dynamical models of the system to conclude that some DM is needed in this galaxy: for various DM halo models, the DM fraction varies between 34 and 45 per cent. They conclude that the most intermediate-luminosity galaxies have very low concentration halos, whereas some high-mass ellipticals have very high concentrations. Also, Napolitano et al. (2011) studied using PN.S the bright elliptical galaxy NGC 4374 (M84) based on ~ 450 velocities together with extended long-slit stellar kinematics interior to $\sim 5R_{\rm e}$. They again used the Jeans models and deduced that this galaxy must possess a massive DM halo, and this case was the first where the PNe dynamics was found to be consistent with a standard DM halo. Recently, the new extended Planetary Nebula Spectrograph (ePN.S) survey was presented in Pulsoni et al. (2018). This survey is based on observations mostly done with the PN.S and consists of catalogs of PNe for 33 ETGs which is presently the the largest survey of extragalactic PNe identified in the halos of ETGs available.

Teodorescu et al. (2010) studied the ETG NGC 821 using a slitless spectroscopy method with the 8.2 m Subaru telescope and its FOCAS Cassegrain spectrograph created a combined sample of 167 PNe. They showed that there is the Keplerian decline of the LOS velocity dispersion. They fitted the declining velocity dispersion profile[21] and concluded that a DM halo in this galaxy may exist but not necessarily. In a subsequent paper, using the same instrumental setup together with the ESO Very Large Telescope unit 1 (Antu) and its FORS2 Cassegrain spectrograph, Teodorescu et al. (2011) analyzed another well-known ETG, NGC 4649 (M60). They created a catalog with kinematic information for 298 PNe in NGC 4649 and found that they support the presence of a DM halo around M 60. Based on an isotropic, two-component Hernquist model they estimated using the Jeans equation (this time explicitly stated) that the contribution of DM within $3R_{\rm e}$ is approximately 50 percent. The same galaxy was analyzed by the same team together with F. de Lorenzi in Das et al. (2011) using the NMAGIC code: they calculated the DM mass fraction ~ 0.39 in NGC 4649 at $1R_{\rm e}$ and ~ 0.78 at $4R_{\rm e}$. Both of the galaxies studied by Teodorescu et al. (2010, 2011) were also analyzed using the GCs (see below).

To conclude this subsection we present the results of Tian and Ko (2016) who used the spherical Jeans equation to produce the MOND models for their sample of ETGs based on their PNe: they combined the PNe data (up to $6-8R_{\rm e}$) and stellar data from the SAURON database of seven elliptical galaxies, including three galaxies found in Romanowsky et al. (2003) with updated data and four other ETGs which have not been analyzed before. Their sample included the following 7 galaxies: NGC 821, NGC 1344, NGC 3379, NGC 4374, NGC 4494, NGC 4697, and NGC 5846. They found using the Jeans equation that the dynamics of the studied ETGs can be well explained by MOND without the need for DM.

4.2. Globular clusters (GCs)

GCs are also very useful as tracers of DM in ETGs and here we briefly discuss cases of several galaxies. In an early work in this field, Mould et al. (1990) obtained optical multislit spectra of two giant ellipticals, M49 and M87, from the Virgo cluster and found that the velocity dispersion profiles of the cluster systems were flat, thus suggesting the existence of an isothermal halo of DM there. Later, Grillmair et al. (1994) studied the radial velocities of 47 GCs in NGC 1399 in another nearby galaxy cluster, the Fornax cluster. Assuming isotropic distribution of the GCs they obtained a lower limit on a globally constant mass-to-light ratio $M/L_B = 79$ and this suggests that the mass-to-light ratio is several times larger than values of mass-to-light ratio determined from the stellar component closer to the core. In Samurović and Danziger (2006) it was shown for the same galaxy that the velocity dispersion decreases between 4 $R_{\rm e}$ and 10 $R_{\rm e}$ and that there is

[21] Although not mentioned in the text, they most certainly relied on solving the Jeans equation, judging from their Fig. 10.

evidence of DM beyond $\sim 3R_\mathrm{e}$ (as shown above based on the observations obtained using long-slit spectra). NGC 1399 was re-analyzed using 790 GCs as tracers of the gravitational potential in Samurović (2016b): the Jeans equation was solved and it was found that in the Newtonian approximation a significant amount of DM is needed and that NFW model with a dark halo fits well the kinematics of NGC 1399. Three MOND models (standard, simple and toy) were also tested and it was found that none of them can provide a fit of the velocity dispersion profile without inclusion of DM.

The ETG NGC4472 (M49) was studied in Côté et al. (2003) where it was shown that the GCs radial velocities and density profiles provide "unmistakable evidence" of a massive dark halo. This object was again studied in Samurović (2012) using the Jeans modeling based on GCs and it was found that the isotropic Newtonian mass-follows-light models without DM cannot provide successful fits at radii larger than $\sim 2R_\mathrm{e}$ and that in the Newtonian approach DM is needed; several MOND models were also tested and it was shown that a successful fit to the velocity dispersion throughout the entire galaxy without DM could be obtained with the isotropic simple and toy models.

The problem of DM in ETG NGC 3379 discussed above was analyzed using GCs in Pierce et al. (2006) who obtained Gemini/GMOS spectra for 22 GCs associated with it and found that their results suggest a constant value of the velocity dispersion (out to ~ 200 arcsec) which implies a normal-sized DM halo, at odds with the conclusions of Romanowsky et al. (2003) described above. This conclusion is, however, not firm since due to possible orbital anisotropies (see above) they could not rigorously determine the dark halo mass. Bridges et al. (2006) analyzed the galaxy NGC 4649 (M60) using Gemini/GMOS to obtain spectra of 38 confirmed GCs there. They found that between $\sim 1.5R_\mathrm{e}$ and $\sim 3.5R_\mathrm{e}$ the velocity dispersion of NGC 4649 remains constant indicating the existence of DM there. Relying on different spherical, isotropic and axisymmetric, orbit-based dynamical models and the X-ray mass profile they inferred the value of the total mass-to-light ratio that shows an increase by a factor of two with respect to the value calculated in the center.

The ETG NGC 4649 (M60) mentioned above was studied using 121 of its GCs in Samurović and Ćirković (2008a) where it was found using the Jeans equation that even beyond $\sim 3R_\mathrm{e}$ there is no need for DM since $M/L_B \sim 7$ with moderate tangential anisotropies can fit the observed velocity dispersion of this galaxy. Also, all the tested MOND models (simple, standard and toy) can also provide successful fits without the need of DM. The same galaxy was also studied in Samurović and Ćirković (2008b) using the tracer mass estimator (TME) approach of Evans et al. (2003) and also X-ray observations: the importance of DM beyond $\sim 3R_\mathrm{e}$ becomes more evident. Both X-ray and TME methodologies predict rapid increase of the total mass which suggests the existence of DM that starts to dominate in the outer regions. Relying on both methodologies it was concluded that in the interior up to $\sim 4R_\mathrm{e}$ the mass-to-light ratio becomes $M/L_B \sim 20$ which implies at least 50 percent of the DM contribution. At $6R_\mathrm{e}$ there is a disagreement between the two methodologies, namely, the estimate based on X-rays ($M/L_B \sim 33$) is higher than that based on TME ($M/L_B \sim 25$). However, the estimates are consistent due to larger error bars in the last bin.

The sample of 15 ETGs was compiled and analyzed in Deason et al. (2012). They estimated their masses out to five effective radii using both PNe and GCs as tracers and found that the fraction of DM within $\sim 5R_\mathrm{e}$ increases with mass. Another ETG, NGC 821, discussed above based on analysis of their PNe, was also analyzed using GCs. In Samurović et al. (2014) it was shown, using the Jeans equation, that DM is not dominant in this galaxy interior within $3R_\mathrm{e} - 4.5R_\mathrm{e}$ thus confirming the earlier results of Teodorescu et al. (2010); it was also found that MOND could correctly fit the observed velocity dispersion of NGC 821. NGC 5128, also discussed above when the PNe data were presented, was reanalyzed using 780 GCs in Samurović et al. (2016a) in both Newtonian and MOND approaches. Given a rather large number of tracers, it was possible to infer that the departures from the Gaussian are not large and are consistent with isotropic distribution. It was found that beyond $\sim 3R_\mathrm{e}$ the velocity dispersion rises from ~ 150 km s^{-1} (in the inner parts) to ~ 190 km s^{-1} (beyond $\sim 3R_\mathrm{e}$) and that the Newtonian mass-follows-light cannot provide a successful fit of the velocity dispersion for the entire galaxy so that the additional, the dark component, is necessary. The mass-to-light ratio in the inner region of $M/L_B \sim 7$, rises to $M/L_B \sim 26$ beyond three effective radii. The NFW model with DM can provide a good fit to the observed velocity dispersion of NGC 5128 and the all three MOND models (standard, simple and toy) provide good fits without the need of DM.

Schuberth et al. (2010) used radial velocities for 289 GCs around NGC 4636, the giant elliptical from the Virgo cluster. The data were obtained with FORS2/MXU at the Very Large Telescope. They performed the Jeans analysis using both stellar and GC data to better constrain the dark halo properties. Relying on various DM models they found that DM is abundant in the Newtonian approach and, since they also used MOND, they found that the dynamics of NGC 4636 is "more or less" consistent with the prediction of the MOND theory.

The significant contribution to the study of DM in ETGs was made by the SLUGGS[22] (SAGES Legacy Unifying Galaxies and GlobularS) survey. Pota et al. (2013) analyzed kinematics for over 2500 GCs of 12 ETGs (from lenticular to large ellipticals), nine of which published for the first time. Two separate GC subpopulations, blue and red, were studied and it was found that the GC kinematics

[22] `http://sluggs.swin.edu.au`

varies with the GC mean colour. The SLUGGS data from Pota et al. (2013) were used in Samurović (2014) in study of 10 ETGs (NGC 1400, NGC 1407, NGC 2768, NGC 3115, NGC 3377, NGC 4278, NGC 4365, NGC 4486, NGC 4494, and NGC 5846) where the Jeans equation together with application of several stellar population synthesis (SPS) models were used to infer the existence and features of DM relying on both Newtonian and MOND methodologies for regions between[23] $\sim 2R_{\mathrm{e}}$ and $\sim 12.50R_{\mathrm{e}}$ (for NGC 4278). Each individual case was analyzed in detail and the existence of a DM halo as well as results of three different MOND models (simple, standard and toy) were discussed. Three cases of anisotropy were taken into account: the isotropic case ($\beta = 0$), tangentially anisotropic case ($\beta = -0.2$), and radially anisotropic case based on numerical simulations. To determine the contribution of the visible stellar component, various SPS models were used and their predictions were compared to the dynamical mass-to-light ratios obtained using the Jeans models. The $B - V$ colors from the HyperLeda[24] database were used: for a given metallicity the mass-to-light ratios were calculated by applying the fitting formulas from Bell and de Jong (2001). In Samurović (2014) seven different models with several initial mass functions (IMFs) were used and the reader is referred to that paper for details. The main conclusions were as follows: i) In Newtonian approach only one ETG (NGC 2768) could be modeled without DM; three galaxies (NGC 1400, NGC 3377, and NGC 4494) show an increase of the total mass-to-light ratio with radius between the innermost and outermost radii, which suggests the existence of DM in their outer parts. NGC 4486 is the only galaxy that needs significant amount of DM even in its inner region (but its effective radius is extremely large, as noted earlier). The remaining five galaxies require a significant amount of DM beyond $\sim 2 - 3R_{\mathrm{e}}$, and the largest mass-to-light ratio was found in NGC 5846 for which $64.2 < M/L_B < 127.4$ beyond $\sim 6R_{\mathrm{e}}$ was established. ii) In MOND four ETGs could successfully be modeled with the stellar mass only: NGC 1400, NGC 2768, NGC 3377, and NGC 4494 (all fast rotators). The following ETGs require DM even in the MOND approach in their outer parts: NGC 1407, NGC 4278, NGC 5846, and NGC 3115 require DM beyond $\sim 3R_{\mathrm{e}}$, whereas NGC 4365 and NGC 4486 require DM beyond $\sim 1R_{\mathrm{e}}$ and $\sim 0.35R_{\mathrm{e}}$, respectively. This need of DM in ETGs diminishes the appeal of MOND thus confirming the assessment of Famaey and McGaugh (2012) quoted in Introduction above. iii) Since the MOND approach cannot provide a successful fit for galaxies that are slow rotators it was noted that there is a hint that the dynamical mass of $1.58 \times 10^{11} M_{\odot}$ separates two classes of ETGs: the galaxies for which the established dynamical mass is below this value can be fitted using MOND with the stellar component only, whereas those objects for which the dynamical mass is above this value require an additional, DM, halo. This is similar to the conclusion of Cappellari (2016) who claims that the usage of IFS showed that there is a link between ETGs and spirals: below a critical mass of $M_{\mathrm{crit}} \approx 2 \times 10^{11} M_{\odot}$ "fast rotator ETGs form a parallel sequence in galaxy properties with spiral galaxies, whereas slow rotators dominate above M_{crit}."

A few more related papers by the SLUGGS team are also mentioned here. In Napolitano et al. (2014) the case of the massive slow rotator ETG NGC 5846 was studied in detail using the GCs from the SLUGGS survey. The authors used the self-consistent Jeans model analysis of red and blue GC subpopulations and included a dispersion-kurtosis analysis to break the degeneracies between DM, anisotropy of orbits and IMF and found that a massive DM halo, consistent with the ΛCDM model, must be present in NGC 5846. In Forbes et al. (2016) the mass of GC systems in ETGs was compared to the host galaxy's DM content on similar spatial scales and a strong correlation between the mass of the blue (metal-poor) GC subpopulation and the DM content within a fixed $8R_{\mathrm{e}}$ was found: for a galaxy with the blue GC fraction of 60 percent the DM fraction of 86 percent is expected over similar spatial scales. They also claim that NGC 4494 possesses a heavy DM halo. In Forbes et al. (2017) a full catalog of approximately 4000 GC radial velocities in 27 nearby ETGs of the SLUGGS project was presented and the data compiled therein provide the opportunity to study the nearby ETGs using various approaches, included the use of the Jeans equations. Finally, in Alabi et al. (2017) a detailed analysis of the DM fraction in ETGs was performed. The authors used GC kinematics data, primarily the SLUGGS survey, to measure the DM fraction and the average DM density within the inner $5R_{\mathrm{e}}$ for 32 nearby ETGs and found a high DM fraction (≥ 0.6) within $5R_{\mathrm{e}}$ in most of their sample. Alabi et al. (2017) also found a tentative evidence that lenticulars, unlike ellipticals, have mass distributions that are similar to spiral galaxies where the DM fraction decreases within $5R_{\mathrm{e}}$ as the galaxy luminosity increases.

In Samurović (2017) kinematics and dynamics of two lenticulars galaxies NGC 1023 and NGC 4526 from the SLUGGS survey were analyzed. Their GCs extend beyond approximately seven effective radii. The Jeans equation was solved in the Newtonian mass-follows-light approach (assuming both a constant mass-to-light ratio and a dark halo in the NFW form) and in MOND. Using the Newtonian approach it was found that NGC 1023 does not need a significant amount of DM while in NGC 4526 DM fully dominates stellar matter in the total dynamical mass. All tested MOND models can provide successful fits of the observed velocity dispersion while in the case of NGC 4526 a hint of a need of DM was detected.

Bílek et al. (2019) recently analyzed a sample of 17 ETGs based on their GCs where nearly

[23] For NGC 4486 an unusually large effective radius, $R_{\mathrm{e}} = 704$ arcsec, was assumed.

[24] http://leda.univ-lyon1.fr

all galaxies have more than 100 archival GC radial velocities measurements available and the GCs systems extend mostly over $\sim 10R_{\rm e}$. This sample is shown in Table 1 which is based on Tables 1 and 5 in Bílek et al. (2019). To this large sample three simulated galaxies obtained from the ΛCDM simulation were added for the purpose of helping the interpretation of the results. In Bílek et al. (2019) the Jeans analysis was employed where the goodness of the fits was rated in several ways: using the χ^2 confidence, the corrected Akaike's information criterion (AICc; Akaike 1974, Hurvich and Tsai 1989) and the logarithm of Bayes's factor (LBF; Jeffreys 1961): it was explored whether dark halos follow the theoretically predicted stellar-to-halo mass relation (SMHR, the relation between the $\log M_*/M_{\rm vir}$ ratio and $\log M_{\rm vir}$, where M_* and $M_{\rm vir}$ are stellar and virial mass, respectively) and the halo mass-concentration relation (HMCR, the relation between the concentration parameter, c and $\log M_{\rm vir}$). It was shown that the positions of all individual galaxies agree with the predictions of both theoretical relations within error bars (see Fig. 3 in Bílek et al. 2019). Also, the RAR relation in the case of massive ETGs was studied and a comparison of ΛCDM and MOND approaches of ETGs was explored in the analysis. The RAR points of ETGs broadly follow the RAR relation established for spiral galaxies but with more scatter and the mean dynamical acceleration is higher at a given Newtonian acceleration (see Fig. 6 in Bílek et al. 2019). The contribution of stellar matter was calculated using the study of Bell and De Jong (2001) relying on the central colors from the HyperLeda database and several SPS models were used (see Section 2.3 in Bílek et al. 2019 for more details). Also, in addition to the isotropic case ($\beta = 0$), the cases of radially anisotropic case given with $\beta(r) = 0.5r/(r + 1.4R_{\rm e})$ from theoretical expectations from merging collisionless systems (Mamon and Łokas 2005) and tangentially anisotropic case with the constant value of $\beta = -0.5$ were also taken into account. The MOND analysis was based on the study of the simple model (Eq. (7)). It was shown that MOND predicts the velocity dispersion profiles in most ETGs better than the ΛCDM model. In Table 1 the fractions of DM inside $1R_{\rm e}$ and $5R_{\rm e}$ for the ΛCDM model for the isotropic case are listed. For the total sample the median contributions of DM inside $1R_{\rm e}$ and $5R_{\rm e}$ are 20 and 72 percent, respectively, which is in agreement with the results found in literature (Cappellari et al. 2011, Alabi et al. 2017). The DM fraction of slow rotators is much higher than that of fast rotators: for the fast rotators the DM fractions are equal to 18 and 69 percent inside $1R_{\rm e}$ and $5R_{\rm e}$, respectively, whereas for the slow rotators

Table 1. Sample of ETGs with GCs

Name	d	$1'$	$\log L$	$R_{\rm e}$	Type	Env	Rot	$B-V$	$f^{\rm DM}_{1R_{\rm e}}$	$f^{\rm DM}_{5R_{\rm e}}$
	[Mpc]	[kpc]	$[L_\odot]$	[kpc]				[mag]	[%]	[%]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
N 821	24^{+2}_{-2}	7.0	10.5	4.7	E6	F	f	0.87	21	71
N 1023	$11.4^{+0.9}_{-0.8}$	3.3	10.5	2.7	S0	G	f	0.91	5	33
N 1399	20^{+2}_{-1}	5.8	10.7	3.5	E1pec	C	s	0.93	15	69
N 1400	26^{+4}_{-4}	7.7	10.4	3.4	S0/E0	G	f	0.89	13	60
N 1407	29^{+4}_{-3}	8.4	11.0	9.4	E0	G	f	0.95	28	84
N 2768	22^{+3}_{-2}	6.5	10.7	8.9	E6/S0	G	f	0.91	32	77
N 3115	$9.7^{+0.4}_{-0.4}$	2.8	10.2	4.8	S0	F	f	0.90	31	81
N 3377	$11.2^{+0.5}_{-0.5}$	3.3	9.9	2.9	E6	G	f	0.82	20	69
N 4278	16^{+2}_{-1}	4.7	10.2	2.5	E1-2	G	f	0.90	11	59
N 4365	20^{+2}_{-2}	5.9	10.7	8.5	E3	G	s	0.93	40	89
N 4472	$16.3^{+0.8}_{-0.7}$	4.7	10.9	3.9	E2	C	s	0.93	19	72
N 4486	16^{+1}_{-1}	4.7	10.8	5.8	E0	C	s	0.92	50	88
N 4494	$17.1^{+0.9}_{-0.8}$	5.0	10.4	3.7	E1-E2	G	f	0.85	13	54
N 4526	17^{+2}_{-1}	4.9	10.4	2.7	S0	C	f	0.89	8	46
N 4649	17^{+1}_{-1}	4.9	10.8	5.1	E2/S0	C	f	0.93	16	69
N 5128	$4.2^{+0.3}_{-0.3}$	1.2	10.5	6.2	S0pec/Epec	G	f	0.87	43	89
N 5846	25^{+2}_{-2}	7.2	10.7	8.1	E0	G	s	0.94	40	88

NOTES: Col. 1: Name of the galaxy. Col. 2: d – Galaxy distance and its uncertainties according to Tonry et al. (2001). Col. 3: $1'$ – The distance corresponding to 1 arcmin if the galaxy is found at the distance of d. Col. 4: L – Galaxy B-band luminosity calculated from d and the apparent magnitude listed in the HyperLeda database. Col. 5: $R_{\rm e}$ – Sérsic effective radius assuming the galaxy distance d. Note that for NGC 4486 this effective radius is equal to 74 arcsecs. Col. 6: Type – Galaxy morphological type. Col. 7: Env – Galaxy environment (F = field, G = group, C = cluster). Col. 8: Rot – Galaxy rotator type ("s" indicates slow and "f" indicates fast rotators). Col. 9: $B - V$ – Color index according to the HyperLeda database. Cols. 10 and 11: DM fraction for the isotropic ΛCDM model at $1R_{\rm e}$ and $5R_{\rm e}$, respectively.

the DM fractions become 40 and 88 percent at the same radii, respectively. The introduction of anisotropies does not significantly change these results: for brevity we only note that, for example, for the tangentially anisotropic orbits ($\beta = -0.5$) the DM fractions for the whole sample are 18 and 70 percent (inside $1R_\mathrm{e}$ and $5R_\mathrm{e}$ respectively). For some galaxies (NGC 1399, NGC 1407, NGC 4278, NGC 4365, NGC 4472, NGC 4486 and NGC 5846) high values of the mass-to-light ratio were calculated, see Table 6 in Bílek et al. (2019)[25] because the observed velocity dispersions are also high. It is worth mentioning the case of two galaxies that could not be fitted well with the MOND models. Both of them are central galaxies in clusters: NGC 1399 (in Fornax) and NGC 4486 (in Virgo). This may lead to the conclusion that MOND needs additional DM contribution in galaxy clusters. The MOND models tended to underpredict the velocity dispersion. Bílek et al. (2019) proposed that it is caused by galaxy interactions that bring the GC systems out of dynamical equilibrium. This is expected since massive ETGs occur in dense environments. We indeed found many signs of recent interactions in the galaxies that were most problematic for MOND.

Finally, we briefly discuss the recent findings of Samurović and Vudragović (2019) related to two massive nearby ETGs, NGC 4473 and NGC 4697[26], both fast rotators. Newtonian (purely baryonic and also models based on the Einasto (1965) DM profile) and MOND models were tested using GCs as tracers of gravitational potential of the galaxies. For the first time, in addition to four MOND formulas previously applied (simple, standard, toy and Zhao (2007)[27]), a set of five rarely used MOND interpolation functions $\mu(a/a_0)$ listed in Famaey and McGaugh (2012) was used. The data from the SLUGGS database from Forbes et al. (2017) were used for the purpose of establishing the profiles of the velocity dispersions of both galaxies. To calculate the contribution of the visible, stellar, component the new 1.40 m "Milanković" telescope mounted at Vidojevica was used to obtain the deep photometry in the B- and V-bands. NGC 4473 was modeled interior to $\sim 12R_\mathrm{e}$ (radius which encompasses approximately 97 percent of the total light of the galaxy) and NGC 4473 was modeled interior to $\sim 3R_\mathrm{e}$ (radius which encompasses approximately 79 percent of the total light of the galaxy) using several SPS models with the Salpeter (1955) and Kroupa (2001) IMFs. It is important to stress that the contribution of the stellar component was not based on the central parts only but was a function of radius (see Fig. 5 in Samurović and Vudragović 2019): the value of the stellar mass-to-light ratio in the B-band varies between approximately 7 in the inner regions to 3 in the outer regions. Three cases of anisotropies were tested and it was found that in most tested cases Newtonian purely baryonic models for both galaxies could not provide a successful fit of the observed velocity dispersion without the additional dark component and that the introduction of the dark Einasto halo improved the quality of the fits. Nevertheless, the solutions for both galaxies based on purely baryonic tangentially anisotropic models with the Salpeter IMF were found. In Fig. 3, the best-fitting results for NGC 4473 (for which the central stellar spectrum was shown in Fig. 1) are plotted. Both plotted Newtonian models were based on the Into and Portinari (2013) SPS model (IP13 model). As regards the MOND models, it was shown that all the tested nine MOND models can successfully fit the two ETGs assuming only the visible stellar component and that no additional DM was needed in any of the tested MOND models. In Fig. 3, the best-fitting MOND results were also plotted for the simple, standard and

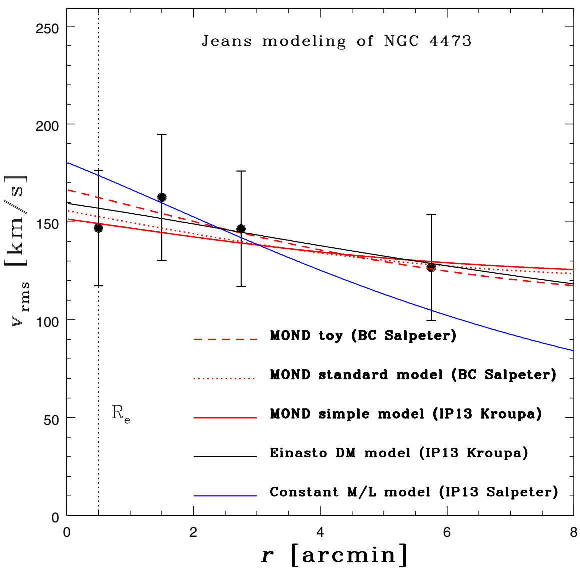

Fig. 3. *The Jeans modeling of the elliptical galaxy NGC 4473. The Newtonian constant mass-to-light ratio model based on the tangentially anisotropic ($\beta = -0.2$) IP13 model with the Salpeter IMF is plotted with the full blue line. The DM model based on the isotropic Einasto dark halo and the stellar component based on the IP13 model with the Kroupa IMF is plotted with the full black line. The three MOND models are plotted with the red lines: the simple model based on IP13 model with the Kroupa IMF is plotted with the full line, the standard model is based on BC model with the Salpeter IMF is plotted with the dotted line and the toy model based on the BC model with the Salpeter IMF is plotted with the dashed line. The vertical dotted line indicates one effective radius. See text for more information.*

[25] See also also Table 7 in Samurović (2014) where different approach also based on the Jeans equation was used.

[26] We remind the reader that NGC 4697 was the subject of one of the earliest applications of the Jeans equation on ETGs in Binney et al. (1990).

[27] Zhao (2007) model was applied for the first time in the literature on ETGs.

toy models (the remaining six MOND models provided fits of similar quality, see Samurović and Vudragović 2019): the simple models are tangentially anisotropic ($\beta = -0.2$) based on the IP13 model with the Kroupa IMF, whereas both standard and toy models are based on the Bruzual and Charlot (2003) model (BC model) with the Salpeter IMF, but the standard model is isotropic and the toy model is made with radially anisotropic orbits.

5. CONCLUSIONS AND OUTLOOK

The year 2019 marks the centenary of the publication of the book *Problems of Cosmogony and Stellar Dynamics* by Sir James Jeans in which he presented the equations that are now known and used as the Jeans equations. In this paper the basic foundations of the Jeans equations were presented together with a brief historical background. It was shown that the Jeans equations remain an important tool in the study of galaxies, especially massive ETGs and the problem of DM in their mass content. The application of the usage of the Jeans equations in massive nearby ETGs in case of integrated stellar spectra in galaxies (both long-slit and integrated field spectra), and various tracers of gravitational potential such as PNe and GCs was discussed.

Although the Jeans equations were widely used in the study of dynamics of ETGs, as evidenced with the DM studies described in the present paper, one should also bear in mind that studies of other types of objects can also be based on the Jeans modeling: we list here the studies of the Milky Way or Milky Way's analogues (see e.g., Battaglia et al. 2005, Samurović and Lalović 2011, Kafle et al. 2018), but also studies of ensemble of galaxies ranging from superclusters (see e.g., McLaughlin et al. 1999), to clusters of galaxies (see e.g., Biviano and Girardi 2003, Mamon et al. 2019) and groups of galaxies (see e.g., Duarte and Mamon 2015). The recent study related to the ultra-diffuse galaxy NGC 1052-DF2 which apparently shows the lack of DM (the inferred mass of the halo is 400 times lower than expected) (van Dokkum et al. 2018a,b) was analyzed using the Jeans modeling formalism from Wasserman (2018) and Emsellem et al. (2018).

Some future work related to DM in ETGs (based on the Jeans equations and other approaches) will include (see e.g., Cappellari 2016): the study of trends in population and kinematics at radii beyond two effective radii, studies of the problem of stellar population and the analysis of IMF in ETGs, studies of trends in total density profiles at large galactocentric distances where DM is expected to dominate, and the comparison with spirals. Another important type of comparison needs to be performed on more individual ETGs: it is necessary to analyze DM using both PNe and GCs together (where possible). The discrepancies noted in some cases, such as NGC 5128 (see Fig. 7 in Samurović 2016a) indicate that different tracers provide different estimates of the total dynamical mass, and thus the uncertain contribution of DM is estimated. This problem needs to be addressed with more dedicated observational studies in the future. Useful catalogues are being created and we mention here the paper by Dabringhausen and Fellhauer (2016) who presented a compilation of 1715 ETGs from the literature, spanning the luminosity range from faint dwarf spheroidal galaxies to giant elliptical galaxies.

A seldom posed question related to DM in ETGs is: what makes the dark, i.e., invisible component in such systems? Apart from the fact that DM matter should be cold (CDM mentioned above, part of the ΛCDM model), not much is known. In Samurović (2016b) in a discussion of DM in NGC 1399 a brief list of possible candidates (more or less successful) for DM in ETGs was given: the candidates include massive neutrinos (note that this is a hot DM candidate, with $m_\nu = 2$eV), warm-hot intergalactic gas as dark baryonic content, sterile neutrino with mass of 11 eV as a candidate for warm DM (WDM) and decaying sterile neutrinos with masses of 7.1 keV and 17.4 keV.

The Jeans equations are shown to be an indispensable tool in the field of galaxy dynamics and it is not a surprise that hundred years since their introduction they are being applied using the latest developments in the computational field. Here we mention the use of the Jeans equations in an attempt to reconstruct the underlying DM mass without the assumptions about the density and anisotropy functions: in Diakogiannis et al. (2019) the Generative Adversarial Networks (GAN), i.e., a powerful class of neural networks used for unsupervised learning, was employed in the attempt to reconstruct nonparametrically the LOSVD of a given galaxy. It was shown that with realistic numerical simulations of dwarf spheroidal galaxies one can distinguish between competing DM distributions and recover the anisotropy and mass profile of the system.

The new "Milanković" 1.40 m telescope mounted at the Astronomical Station Vidojevica has proven to be a very useful tool in obtaining images of nearby ETGs thus providing the information on the stellar content inside several effective radii. Future observations will provide data on more ETGs which will enable us to analyze larger samples of nearby ETGs relying on ever-improving SPS models. Thus the contribution of the elusive DM in ETGs will hopefully be better understood and the problem of DM in galaxies will eventually be solved, although it seems that a long road is still ahead of us.

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ТАМНА МАТЕРИЈА У МАСИВНИМ ГАЛАКСИЈАМА РАНОГ ТИПА: 100 ГОДИНА ЏИНСОВИХ ЈЕДНАЧИНА

S. Samurović

Astronomical Observatory, Volgina 7, 11060 Belgrade 74, Serbia
E–mail: *srdjan@aob.rs*

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Прегледни рад по позиву

Џејмс Џинс (James Jeans) је 1919. године објавио књигу *Проблеми космогоније и звездане динамике* (*Problems of Cosmogony and Stellar Dynamics*) у којој је сумирао свој рад на динамици звезданих система заснован на радовима које је објављивао од 1915. године. Како би се обележила стогодишњица штампања ове књиге овде приказујемо једну примену његовог рада која је релевантна за савремена истраживања галаксија: анализирамо проблем тамне материје у масивним галаксијама раног типа (елиптичним и сочивастим галаксијама) користећи различите доступне посматрачке податке. После дискутовања основних поставки Џинсових једначина проучавамо њихову примену на интегрисане звездане спектре у галаксијама, податке везане за планетарне маглине и, посебно, податке везане за глобуларна јата. Да би се одредио допринос тамне компоненте у галаксијама раног типа коришћено је неколико модела синтезе звезданих популација. Показано је да тамна материја не доминира у унутрашњим областима галаксија раног типа, али да постаје све значајнија у областима изван три ефективна радијуса.