Serb. Astron. J. № 188 (2014), 55 - 65
DOI: 10.2298/SAJ1488055R

UDC 52–334.2
Original scientific paper

RELATIVISTIC LENGTH AGONY CONTINUED

D. V. Redžić

Faculty of Physics, University of Belgrade
PO Box 44, 11000 Beograd, Serbia
E–mail: *redzic@ff.bg.ac.rs*

(Received: April 13, 2014; Accepted: May 7, 2014)

SUMMARY: We made an attempt to remedy recent confusing treatments of some basic relativistic concepts and results. Following the argument presented in an earlier paper (Redžić 2008b), we discussed the misconceptions that are recurrent points in the literature devoted to teaching relativity such as: there is no change in the object in Special Relativity, illusory character of relativistic length contraction, stresses and strains induced by Lorentz contraction, and related issues. We gave several examples of the traps of everyday language that lurk in Special Relativity. To remove a possible conceptual and terminological muddle, we made a distinction between the relativistic length reduction and relativistic FitzGerald–Lorentz contraction, corresponding to a passive and an active aspect of length contraction, respectively; we pointed out that both aspects have fundamental dynamical contents. As an illustration of our considerations, we discussed briefly the Dewan–Beran–Bell spaceship paradox and the 'pole in a barn' paradox.

Key words. **relativistic processes**

1. INTRODUCTION

Recently, Franklin (2010) published a thought-provoking and curious didactic paper on the Lorentz contraction and related issues. While his intention was 'to correct students' misconceptions due to conflicting earlier treatments', I believe that the paper could be a confusing reading for the student. It is a hardly extricable and certainly challenging mixture of truths, half-truths and erroneous statements about some basic relativistic concepts and results. Thus Franklin's argument may sound correct to the unexperienced ear. The situation is aggravated by the circumstance that some of his contentions are in conformity with interpretations found in some authoritative books on the subject.

In another didactic paper, McGlynn and van Kampen (2008) contend that the phenomenon of different values of charge densities in a current-carrying wire as measured by observers in different inertial frames, due to relativistic length contraction, is an effect 'which perfectly demonstrates "the pole in a barn" paradox' at room velocities. I think, however, that the authors are wrong here, namely, two distinct aspects of relativistic length contraction are exemplified in the two situations.

Neither paper is an exception. It is a notorious fact that understanding relativity is a painful, nay agonizing process. Already at a first step towards the conquest of relativistic mentality, one strikes the hard wall of everyday language. As Schrödinger (1977) pointed out, 'everyday language is prejudicial in that it is so thoroughly imbued with the notion of time — you cannot use a verb (*verbum*, "the" word, Germ. *Zeitwort*) without using it in one or the other tense. [...] [Special Relativity] meant the dethronement of time as a rigid tyrant imposed on us from outside, a liberation from the unbreakable rule of "before and after"'. More recently, Mermin (2003) argued that 'to deal with relativity one must either critically reexamine ordinary language, or abandon it altogether'. The present author believes that alas it is impossible to abandon altogether the metalanguage of everyday speech. Physical meaning is unavoidably blurred by linguistic meaning and *vice versa*. (Incidentally, the word 'agony' need not necessarily have a painful connotation; choosing the title for this paper, I had also in mind its Greek sense ('a contest for victory').)

As an illustration of the conceptual and terminological traps that lurk in Special Relativity, recall the old duel between Dingle (1962) and Born (1963) about the reciprocity of time dilatation which aroused a prolonged controversy in the *Nature*. In my opinion, Born won. Briefly, Dingle has made the same kind of error the student usually makes: two different quantities are denoted by one and the same symbol and thus confused. (A compound event that takes place at various spatial points of an inertial frame K (corresponding to the motion of a clock at constant velocity $\boldsymbol{v}$), and has a duration of $1/\sqrt{1-v^2/c^2}$ K-seconds, and a compound event that takes place at one spatial point of the same K-frame and has the same duration of $1/\sqrt{1-v^2/c^2}$ K-seconds must not be identified; those events are two distinct straight lines in Minkowski space-time (cf. Redžić 2006a).) As another illustration, note that a natural inference that the final outcomes of events must be the same with respect to two inertial observers cannot be generalized to two infinite continuous sequences of inertial observers, a lesson the present author has fully learned only recently (Peregoudov 2009, Redžić 2009). Perhaps most surprisingly, even the meaning of the Lorentz contraction, which is generally accepted to be the simplest relativistic phenomenon, is hard to grasp and becomes the stumbling block in various contexts (Zapolsky 1988, Cavalleri and Tonni 2000, Redžić 2008a).

This is small wonder. As the first physical consequence of the Lorentz transformations, the student learns that the length of a rod which is uniformly moving along itself with velocity $\boldsymbol{v}$ is reduced by a factor $\sqrt{1-v^2/c^2}$ compared with the rod's length as measured in its rest frame. While the phenomenon has been dubbed 'relativistic length contraction', the student is immediately warned 'but of course nothing at all has happened to the rod itself'. However, the term 'contraction' connotes shrinking ('cold contracts metals'), shrinking connotes change, and 'change' in physics involves something happening; what happens *before* and *after* is measured in one and the same reference frame. Thus, learning that the rod contracts, yet nothing has happened to it, the student strikes the hard wall of everyday language. Moreover, even some excellent books on Special Relativity explain that '[...] the different measures of length are intimately connected with the lack of absolute simultaneity' (Rosser 1964), or, in the same vein, '[...] the contraction, when we observe it, is not a property of matter but something inherent in the measuring process' (French 1968).

In a recent paper, *Towards disentangling the meaning of relativistic length contraction* (Redžić 2008b), I pointed out that such interpretations confuse derivation of the phenomenon and its root, thus adding a conceptual problem to the terminological one. (Henceforth, I refer to this paper as the *Contraction*.) I argued that there is also a fundamental active aspect of relativistic length contraction: a rod initially at rest in an inertial frame, after a constant velocity $\boldsymbol{v}$ is imparted to it so that it moves freely and uniformly along itself, is contracted (its length is reduced), all with respect to that frame; the phenomenon is due to acceleration of the rod relative to that frame, and is described by the well-known formula, under the proviso that the acceleration was rest length–preserving in the final outcome. I inferred that without the active aspect of length contraction, i.e. without the rest length–preserving accelerations, there is no Special Relativity. The Lorentz transformation, even the formulation of the principle of relativity, is built on the active aspect. Thus, the Lorentz transformations have dynamical contents which are encrypted in their received Minkowskian (geometric) interpretation.

It appears that Franklin (2010) and McGlynn and van Kampen (2008) did not notice the *Contraction*, or they chose to ignore it. (As the ubiquitous worm of doubt reminds me, there is a possibility that *my* analysis of the length contraction phenomenon was wrong; however, I was unable to find any flaws in my argument.[1]) In the present paper I will point out what I find to be the weak points of the recent didactic papers; the same fallacies appear in some authoritative references dealing with interpretational aspects of length contraction. Since the importance of this venerable issue cannot be overestimated in teaching Special Relativity and related areas such as Particle Physics and Astrophysics, the present analysis, hopefully, should be instructive for the students.

2. THE LENGTH OF A MOVING ROD REVISITED

In this section, for the convenience of the reader I will summarize the main conclusions of the *Contraction*, somewhat sharpening the terminology.

2.1. The relativistic length reduction

Consider two inertial reference frames S and S' in standard configuration, S' is uniformly moving at speed v along the common positive x, x'-axes, and the y- and z-axis of S are parallel to the y'- and z'-axis of S', respectively. Take a solid rod parallel to the x, x'-axes, at a permanent rest with respect to

[1]Note an obvious spot of sloppy writing in the *Contraction*, p 198: text 'because of being accelerated relative to S' should be replaced by 'because of having been accelerated relative to S'; I thank Vladimir Hnizdo for spotting that omission.

the S' frame, and let l'_0 be the length of the rod as measured in S' by a given measuring rod also at rest in S'. What is the length of the rod as measured in S employing the same measuring rod which is now at rest in S?

Following Einstein's (1905) prescription for ascertaining 'the length of a uniformly moving rod [...] in the "resting" frame S', by using the Lorentz transformation, one deduces in the well-known way that the length of our uniformly moving rod as measured in S, l_v, is given by

$$l_v = l'_0\sqrt{1 - v^2/c^2}\,. \qquad (1)$$

The phenomenon expressed by Eq. (1), that *one and the same rod* has different lengths l_v and l'_0 as measured in the S and S' frames, respectively, where S' is the rest frame of the rod, in what follows I will call the relativistic length reduction. Eq. (1) is usually called 'relativistic length contraction' or 'conventional length contraction' (Styer 2007); it seems that Torretti (2006) was the first to point out that 'contraction' is a misnomer when applied to Eq. (1). The term 'reduction' seems to be more appropriate here, as being perhaps more neutral than 'contraction', which connotes shrinking in one frame, as was pointed out above. (I am grateful to Professor Giuliano Boella for stimulating correspondence concerning this terminological point.) Of course, the contents of Eq. (1) are not exhausted by the length of a moving rod, cf. footnote 6 of the *Contraction*. Note that the term 'one and the same rod' used above has a peculiar, special relativistic meaning: it connotes that no action was exerted upon the rod by a mere different choice of inertial reference frame (observer), and yet the rod does not have the same length in the various frames (corresponding to various cross-sections of the world-strip of the rod, cf. the appendix). Again, everyday language is *the* problem in Special Relativity.

Einstein's well-known prescription for ascertaining the length of a moving rod is usually presented in the literature as a 'very natural operational definition'. However, the term 'definition' can be misleading; it could imply that perhaps some other definition, leading to a different value of length of a uniformly moving rod, could be legitimately introduced. (As Dieks (1984) pointed out, the term 'definition' '[...] possesses the connotations of arbitrariness and conventionality'.) As far as I understand Special Relativity, Einstein's length of a moving rod is the length of a segment of a stationary line taken up instantaneously by the moving rod, all with respect to S. What else on earth could be the length of a moving object? Measuring that *stationary* length would hardly be a 'measurement of a particular kind'. (However, to ascertain the stationary line segment would require, e.g. taking a photograph of the moving rod and a clever interpretation of the photograph (Kraus 2008).) Therefore, it could be perhaps somewhat misleading to state that Einstein's length of a moving rod 'does refer to measurements of a particular kind [...]' (French 1968, p 152). Einstein's 'operational definition' is not so much a *measurement* of a particular kind but rather a perfectly classical explanation of what is the length of a moving rod (regardless of how it moves), whose only peculiarity is that one instant of the S-time should be understood according to Special Relativity.

2.2. The relativistic FitzGerald–Lorentz contraction

Now in the Relativity Paper, Einstein stated that if the same rod to be measured is at rest in S, then 'according to the principle of relativity' its length as measured in S, l_0, must be equal to l'_0,

$$l_0 = l'_0\,, \qquad (2)$$

employing of course the same measuring rod as in the earlier measurements. Eqs. (1) and (2) imply

$$l_v = l_0\sqrt{1 - v^2/c^2}\,. \qquad (3)$$

Eq. (3) relates the length l_0 of the rod at rest to its length l_v when it is in uniform motion along itself at the speed v, all with respect to the inertial frame S. The phenomenon described by Eq. (3) in what follows I will call the relativistic FitzGerald–Lorentz contraction, or shortly the FitzGerald–Lorentz contraction. Obviously, in this case there is a change of the rod relative to S (its length has changed); the change is due to acceleration of the rod from rest to the state of uniform motion. [Basically, considering the rod as an atomic configuration, two distinct stationary configurations (described by *statics*, not kinematics) leading to two distinct lengths, are established in the two states of motion of the rod.] This is contrary to the situation described by Eq. (1), in which case there is no change of the rod in the standard physicists' sense of the word (involving alterations in the rod with time in one and the same *inertial* reference frame); only the frame S world-map is substituted for the frame S' world-map, without exerting any action upon the rod itself. It seems natural to call the content of Eqs. (1) and (3) a passive and an active interpretation of relativistic length contraction, respectively.

Two words of caution are needed here.

First, one should make a clear distinction between the rest length Lorentz-invariance, which is a truism, and the rest length frame-independence, which is a fundamental physical assumption. Namely, if a rod is uniformly moving along itself with respect to an inertial frame, making a Lorentz boost to its rest frame S^*, one can measure its rest length l^*_0. The quantity l^*_0 is *a fortiori* Lorentz invariant. This of course means, on the basis of Eq. (1), that $l^*_0 = l_{v_1}/\sqrt{1 - v_1^2/c^2} = l_{v_2}/\sqrt{1 - v_2^2/c^2}$, etc, where $v_1, v_2, ...,$ are speeds of the rod relative to inertial frames $S_1, S_2, ...,$ respectively (all the frames being in standard configuration with S^*). On the other hand, Einstein's assumption (2) has quite a different meaning, which was perhaps most clearly expressed by Resnick (1968, p 93): 'The *rest length* of a rod is an absolute quantity, the same for all inertial observers: If a given rod is measured by different inertial observers by bringing the rod to rest in their respective frames, each will measure the same length.' This rest length frame-independence could be also termed the absolute Lorentz-invariance of rest length.

Second, while Eq. (3) is an obvious consequence of Eqs. (1) and (2), one point should be stressed. Namely, according to Einstein (1905), a rod at rest with its axis lying along the x-axis, having the length l_0, after 'a uniform motion of parallel translation (with velocity v) along the x-axis [...] is imparted to the rod' will have the length l_v given by Eq. (3), all with respect to the same inertial frame S. This is so regardless of the way the speed v was imparted to the rod (Einstein made no restrictions). Thus, according to Einstein, an *arbitrary* acceleration of an arbitrary rod, starting from rest, with the only proviso that the acceleration leads eventually to a uniform (unconstrained) motion of the rod along its length, in a persistent internal state, does not (eventually) change the rest length of the rod.

2.3. Discussion

There is a long tradition, originated by Einstein (1905), to present relativistic length contraction as a purely kinematical effect. As we pointed out in the *Contraction*, 'the fascinating simplicity and universality of Einstein's derivation of relativistic length contraction was a kind of red herring, without virtually anybody being aware of that [...]'. Thus, in the usual textbook presentations of Special Relativity, the active interpretation of length contraction is either neglected or introduced tacitly. For example, after a brief discussion of the relativistic length reduction, Eq. (1), as a kind of a *velocity perspective* effect ('but of course nothing at all has happened to the rod itself'), Rindler (1991) stated that the phenomenon 'is no "illusion": it is real and, in principle, usable'. However, giving an argument for the last statement, as is clear from the context, he was tacitly assuming Eq. (2), i.e. he had tacitly passed from the passive to the active interpretation of length contraction. Rindler is no exception. For some mysterious reason, the pride of place has been given to the passive interpretation by various authors including Einstein, Born, Pauli. The heuristic level of Special Relativity, 'helping us to recognize a great miracle of the world', seems to be kept *sub rosa*.

However, without the active interpretation there is no Special Relativity as a physical theory which 'asserts definite properties of real bodies'. This follows from Einstein's original definition of two inertial frames in standard configuration (which conceptually precedes the formulation of the principle of (special) relativity and a derivation of the Lorentz transformation and which, as far as I am aware, cannot be replaced by another definition), and from a related Einstein's assumption of 'the boostability of rulers and clocks' (Brown 2005), made explicit by Born (1965), cf. footnotes 4 and 12 of the *Contraction*. As was pointed out above, the active interpretation involves changes.

Thus, if there were no change in the (macroscopic) object in Special Relativity, then Special Relativity would not exist as a valid physical theory (it could not even be formulated). However, changes which appear in Special Relativity may have curious properties, requiring a thorough reexamination of everyday language. For example, the FitzGerald–Lorentz contraction described by Eq. (3) possesses the following peculiarity: a free rod in uniform motion along its length is contracted (shrunk) with respect to the S frame and yet it is perfectly relaxed (with no stress relative to both S and S' frames), the contraction being its natural state when it is in that state of motion (all this under the proviso that the rod was unstressed when initially it was at rest in S). Also, contrary to what was sometimes stated in the literature, the contraction is not due to the relative motion of a body; it is due to acceleration (or deceleration, in the reciprocal case of elongation) of the body relative to an inertial frame.

My key inference in the *Contraction* seems to be that a weaker assumption than Einstein's original 'boostability of rulers and clocks' is sufficient for foundation of Special Relativity. The weaker assumption, which I called 'the universal boostability assumption', states that it is possible to boost a measuring rod or clock in a way which leaves their measuring capacity untouched. As far as I am aware, this implies that the rest length of a rod need not be preserved under arbitrary boosts. There is no guarantee of the absolute Lorentz-invariance of rest length.

Finally, one important point should be stressed in relation to the passive interpretation of length contraction. Throughout the present article I insisted that, in accord with the received interpretation of Special Relativity, no action was exerted upon the rod by a mere transition to another inertial reference frame (and that consequently there is no change of the rod in the standard physicists' sense of the word). This is of course true. It appears, however, that there is a natural dynamical content of the relativistic length reduction; this intriguing point, the neglect of which may lead to various misconceptions (a perfect example is the Bell spaceship paradox discussed in subsection 3.5.), will be expounded in a forthcoming paper.

3. SOME FALLACIES IN THE EXISTING LITERATURE

3.1. There is no change in the object in Special Relativity

Franklin based his discussion of length contraction in Special Relativity on the following premise: In Special Relativity, there is no change in the object. It is only the reference frame that is changed from S to S'. Now since that premise runs as a common thread through various authoritative discussions of the topic, it perhaps deserves some clarification.

As a representative example, I choose the famous book *Einstein's Theory of Relativity* by Max Born (1965). In a section under catchy title *Appearance and Reality*, Born pointed out that some opponents of Special Relativity assert that Einstein's theory implies '[...] a *violation of the causal law.* For if one and the same measuring rod, as judged from the system S, has a different length according to its being at rest in S or moving relative to S, then, so these people say, there must be a cause for this change. But Einstein's theory gives no cause; rather it states that the contraction occurs by itself, that it is an accompanying circumstance of the fact of motion.' Born defended Special Relativity by arguing that the opponents have '[...] a too limited view of the concept "change"'. He explained that 'the standpoint of Einstein's theory about the nature of the contraction is as follows: A material rod is physically not a spatial thing but a space-time configuration. Every point of the rod exists at this moment, at the next, and still at the next, and so on, at every moment of time. The adequate picture of the rod under consideration (one-dimensional in space) is thus not a section of the x-axis but rather a strip of the x, ct-plane [parallel to the ct-axis] [...] The "contraction" does not affect the strip at all but rather a section cut out of the [corresponding] x-axis. It is, however, only the strip as a manifold of world points (events) which has physical reality, and not the cross-section. Thus the contraction is only a consequence of our way of regarding things and is not a change of physical reality. Hence it does not come within the scope of the concepts of cause and effect'.

It is clear that Franklin's premise concurs with Born's explanation: there is no change in the object in Special Relativity. It is also clear that Born and Franklin would be right if their arguments referred only to the relativistic length reduction, described by Eq. (1). However, if Eq. (1) were the whole contents of length contraction, i.e. if there were no change in the object in Special Relativity, then Special Relativity would not exist as a physical theory, as is pointed out in the preceding section (cf. also the *Contraction*).

To do justice to Born, it should be noted that in the first part of section *Appearance and Reality*, introducing his 'principle of the physical identity of the units of measure', he essentially argued for a change in the object in Special Relativity (namely, that Eq. (1) and assumption (2) imply the physical validity of Eq. (3)). Unfortunately, in the sequel he confused the (relativistic) FitzGerald–Lorentz contraction (where change is obvious) with the relativistic length reduction (where there is no change in the usual physicists' sense of the word), ascribing properties of the second phenomenon to the first one. Thus his defense of Special Relativity failed.

In *Appearance and Reality*, Born switched several times, tacitly and obviously unconsciously, between the active and the passive interpretation of length contraction, using the same term 'contraction' for both phenomena, and confusing their meanings. This section of Born's book (1965, pp. 251-262) is perhaps a perfect example of how unavoidably terminological confusion leads to conceptual confusion. This confusion seems to be commonplace in the literature. Thus Pauli (1958) in his discussion of the Lorentz contraction confused Eqs. (1) and (3), obviously assuming tacitly Eq. (2). While dealing only with the relativistic length reduction, he ended the discussion with the following query which clearly refers to the active aspect of the phenomenon: 'Should one then ... completely abandon any attempt to explain the Lorentz contraction atomistically? We think that the answer to this question should be No' (Pauli 1958, pp. 11-15).

3.2. There is only one length: the 'rest frame length'

As was pointed out above, Franklin's (2010) premise is that there is no change in the object in Special Relativity. However, since properties like length undergo changes, the author cut the Gordian knot as follows: the Einstein length of a moving object is not a physical attribute of the object! Only its 'rest frame length' is a physically reasonable attribute–length of the object. Moreover, discussing the relativistic length reduction, Eq. (1), as a *velocity perspective* effect, he inferred that 'the "shortening" of a stick that is rotated in four dimensions by a Lorentz transformation is [...] illusory'. However, stating that the relativistic length reduction is an illusion would represent a falsification of Special Relativity.

Now one of Franklin's starting assertions, that 'the measured length of a moving object depends on the "particular way" in which it is measured', is perfectly correct. Indeed, one and the same moving object may have various measured lengths, depending on which *definition* (i.e. which procedure of measurement) of the length of a moving object is being used, all with respect to one and the same reference frame. However, for some reason Franklin ignored the fundamental fact that according to Einstein's Special Relativity the moving object has only one length in the (stationary) frame S, that obeying Eq. (1); it is the only physical reality (world-map) for the S-observer. A photograph of a (small) moving object would indeed be identical to a photograph of an object that is somewhat rotated, but of the same shape and dimensions as compared with the moving object in its rest frame, under the proviso that the rotated object is at rest, as Franklin recalled. However, as is well known, that inference is reached assuming Special Relativity which means, *inter alia*, that the moving object has only one length with respect to S, i.e. that the relativistic length reduction had taken place (cf. French 1968, pp. 150–152, Rosser 1964, pp. 163-168, Gamow 1961, Krauss 2008). What Franklin characterizes as 'the belief that a moving object has a different length', is the only physical reality for the S-observer; the 'belief' is obviously built into the standard Lorentz transformations as $x'\sqrt{1-v^2/c^2} = x - vt$, i.e. in the more familiar form

$$x' = \gamma_v(x - vt)\,.$$

On the other hand, it is clear that Franklin (2010) does assume the validity of the Lorentz transformations. Thus his argument appears to be self-contradictory. Contrary to Franklin's statement, one and the same moving rod has infinitely many lengths in infinitely many inertial frames in the standard configuration with its rest frame S', respectively. According to Special Relativity, none of the lengths is less or more *physically* real than the rod's rest length l'_0; each inertial observer possesses her or his perfectly legitimate physical reality.

Recall that at the end of his famous paper Terrell (1959) stressed that none of his statements there 'should be construed as casting any doubt on either the observability or the reality of the Lorentz contraction [i.e. the relativistic length reduction], as all the results given are derived from the special theory of relativity.' It is perhaps worthwhile to mention here that, analyzing in 1905 how the shape of a body depends on the reference frame in which it is measured, Einstein occasionally used the verb 'betrachten'. This German verb has two meanings: first, to observe, to see, and second, to consider, depending on the context. Various English translators of the Relativity Paper seem to be unanimous that Einstein used 'betrachten' in the first sense. (The present author shares this point of view.) With hindsight, we know today that Einstein (and translators) should have been using *to consider*, or perhaps better *to measure* ('messen', in German), instead of *to observe*, *to view*. (Here of course *to measure* is used in the sense of Einstein's 'operational definition'.) The moral of the story seems to have been known to Democritus: things are not found where their picture is.

[Incidentally, note that the *invisibility* of the FitzGerald–Lorentz contraction implies, *inter alia*, that cartoons in George Gamow's wonderful book *Mr Tompkins in Wonderland*, first published in 1940, are wrong. Of course, one can remedy this by *defining* 'Tompkins' Wonderland' as a place where one sees what is physically measurable, as opposed to what is seen by means of the usual optical equipment such as eyes and cameras, as Dewan (1963) pointed out. By the way, the same remark applies to figure 3 below.]

Finally, comment on another Franklin's (2010) basic assertion that two different inertial frames are required 'in order to compare the measured length of a moving object to its measured length in a system in which it is not moving'. This is of course true in the case of the relativistic length reduction, described by Eq. (1). However, if that were the whole contents of Special Relativity, then it would not exist as a physical theory. As was pointed out in the *Contraction* and also above, the foundation of Special Relativity requires the rest length–preserving accelerations (and also, more generally, it requires the rest properties–preserving accelerations (cf. Redžić 2005, 2006b)). In the case of such *gentle* accelerations (which are *sine qua non* for Special Relativity) one inertial frame would be enough for a comparison of the two lengths.

3.3. Who contends stresses can be induced by Lorentz contraction

Franklin (2010) stated that Lorentz contraction (by which he obviously meant the relativistic length reduction, Eq. (1)) could not induce strains and stresses. He illustrated this with a simple example of a brittle wine glass at rest on a table, pointing out that moving past the wine glass at constant velocity (and looking at it) could not shatter the wine glass. This is of course true: an object at permanent rest and perfectly relaxed in the S' frame, is perfectly relaxed also relative to the S frame (no action was exerted upon the object in the change of the inertial reference frame from S' to S).

Now Franklin also stated that some well known references (FitzGerald 1889, Lorentz 1892, Dewan and Beran 1959, Bell 1976) contended that stresses and strains could be induced by Lorentz contraction.

However, as far as I can see, there is no hint of such a contention in FitzGerald's five-sentence letter to *Science*, where he had suggested a hypothesis that 'the length of material bodies changes, according as they are moving through the ether or across it, by an amount depending on the square of the ratio of their velocity to that of light' (FitzGerald 1889, cf. also Brush 1967). [In my opinion, no distortion or deformation, and thus no stresses, are implied in the following FitzGerald's (1889) sentence: 'We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently.' (It seems to me that the only way of finding stresses in the FitzGerald sentence would be to read Lorentz's ideas into it.) However, I agree with Brown (2005, p. 51) that the FitzGerald supposition was prompted by Heaviside's result for the electromagnetic field of a point charge in uniform motion relative to the ether.] Also, despite appearances, no such contention is the *essence* of Dewan and Beran's and Bell's papers. Namely, Bell clearly stated that '[...] the artificial prevention of the natural contraction imposes intolerable stress' (1976), where by 'the natural contraction' he obviously meant the *relativistic* FitzGerald–Lorentz contraction, described by Eq. (3). (Bell nowhere stated that 'the natural contraction' itself induces stresses.) On the other hand, it is true that Dewan and Beran (1959) described their *Gedankenexperiment* as a demonstration 'that relativistic contraction can introduce stress effects in a moving body'. However, the authors were somewhat sloppy in their wording, as is often the case in discussions dealing with Special Relativity; I think it is clear from the contents of their paper that their *intended meaning* is perfectly summarized by the above quotation from Bell. Thus it seems that only Lorentz (Lorentz et al. 1952, pp. 5–7, 21–23, 27–28) spoke explicitly about deformation (and thus about stresses) of a body in connection with his 'by no means far-fetched' hypothesis that if to a system Σ' of particles in the equilibrium configuration, at rest relative to the ether, 'the velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ is imparted, it will *of itself* change into the system Σ [which is got from Σ' by the deformation $(\frac{1}{\beta l}, \frac{1}{l}, \frac{1}{l})$, where $\beta = (1 - v^2/c^2)^{-1/2}$ and l is a numerical factor allowing for a change in the y and z directions]. In other terms, the translation will *produce* the deformation $(\frac{1}{\beta l}, \frac{1}{l}, \frac{1}{l})$'. (Note that Lorentz subsequently demonstrated that $l = 1$ (Lorentz et al. 1952, p. 27).)

Perhaps the Lorentz wording that bodies 'have their dimensions changed by the effect of translation' prompted Minkowski to characterize the hypothesis as sounding 'extremely fantastical, for the contraction [...] [is to be looked upon] simply as a gift from above,—as an accompanying circumstance of the circumstance of the motion' (Lorentz et al. 1952, p. 81). With the benefit of hindsight, and taking the liberty of rectifying FitzGerald and Lorentz, I believe that both eminent physicists were victims of the traps of ordinary language: concerning their statement that bodies are changed by their translational motion relative to the ether, I think that their intended meaning was that bodies are changed by their acceleration relative to the ether from rest until reaching a steady velocity.

3.4. The original FitzGerald–Lorentz contraction and its relativistic counterpart

Franklin's (2010) starting statement is that 'Lorentz contraction [introduced by FitzGerald and Lorentz] is not what actually occurs for a moving body in special relativity'. While that statement is certainly correct, it seems that the author in the sequel ignored the fact that there is a perfectly legitimate FitzGerald–Lorentz contraction in Special Relativity. This point perhaps needs some clarification.

As is well known, FitzGerald and Lorentz introduced the contraction (shrinking) of bodies in motion relative to the ether. Thus a rod at rest on the earth may be contracted, depending on the direction of its motion through the ether, compared (measured) with the same rod at rest in the ether. However, in the world–map of the FitzGerald and Lorentz, a rod at rest in the ether is not contracted in comparison with an identical rod which is brought to rest relative to the earth; rather, the former rod may be elongated compared with the latter. In this sense, the contraction introduced by FitzGerald and Lorentz is absolute, there is no reciprocity in it. As Franklin pointed out, applying this originally introduced FitzGerald–Lorentz contraction to a variant of the Michelson–Morley apparatus would lead to a *positive* result (cf. Panofsky and Phillips 1955, p. 236). (Note, however, that that conclusion is based on the premise that the velocity of light is the same in all directions only in the ether frame, contrary to Franklin's assertion.)

Now a clear distinction should be made between the original FitzGerald–Lorentz contraction and its relativistic counterpart (which is also called—and justly so—the FitzGerald–Lorentz contraction). Namely, what the two conceptions have in common is shrinking (which I think is due to acceleration); however, contrary to the former, the latter actually occurs for an object to which a constant velocity is imparted in *any* inertial frame (under the proviso of the rest length–preserving accelerations). (Recall that there is no ether frame in Special Relativity simply by virtue of Ockham's razor (for an interesting argument cf. Mirabelli 1985).)

3.5. The Bell spaceship paradox

Section 2 entitled 'The Bell spaceship paradox' seems to be the most mischievous part of Franklin's (2010) paper. While the author asserts that he presents 'the nexus of the Bell spaceship paradox as originally presented by John Bell', actually this is not so. Namely, Bell (1976) took into account the Evett and Wangsness (1960) correction of the original Dewan–Beran formulation of the problem. Thus, instead of connecting the tail of the front spaceship (B) and the nose of the back spaceship (C) as is supposed in Dewan and Beran (1959) and Franklin (2010), in the Bell formulation a thread connects the *corresponding points* of ships. (For the convenience of the reader, the correct Bell setup is depicted in Figs. 1–3.)

Fig. 1. *Three small spaceships A, B and C at rest relative to an inertial frame S, with B and C equidistant from A. At one moment, two identical signals from A are emitted towards B and C.*

Fig. 2. *On reception of these signals, the motors of B and C are ignited simultaneously with respect to S, and the ships accelerate identically along the straight line connecting them.*

Fig. 3. *As B and C accelerate, the thread spanning between the midpoints of the ships before the motors were started travels with them, keeping its initial length. Will the thread break?*

This point is specially clear in the 'mild' variant of the problem, in which at an instant of the S time the ships' acceleration ceases and they coast with the same constant velocity, as measured in the S frame (Redžić 2008a). Namely, assuming the rest length–preserving accelerations of *ships* in the final outcome, ships will eventually FitzGerald–Lorentz contract according to Eq. (3), and thus the final distance between the tail of B and the nose of C will be greater than their initial distance, all with respect to S. It is perhaps worthwhile here to clarify the standard assumption that the thread connecting ships in no way affects the *motion* of ships. Namely, this does *not* mean that the thread does not affect *ships* (it does, cf. Cornwell 2005); instead, that means that the work programmes of the ships' motors are being constantly re-adjusted so as to provide that ships have identical accelerations with respect to S.

The wrong setup apart, Franklin's analysis of his version of the Dewan–Beran–Bell problem (which is obviously the 'tough' variant, the ships' acceleration never ceases) is basically incorrect: contrary to Franklin's repeated statements, there is no common rest frame S' for both ships ('even for continually accelerated spaceships'), as is clear from the corresponding Minkowski diagram, cf. footnote 7 of Redžić (2008a). (Events that are simultaneous in the S frame are not simultaneous with respect to any other frame, and *vice versa*.) Consequently, there is no rest frame distance between ships (there is no frame in which both ships are *simultaneously* at rest, except of course the S frame at $t = 0$) and Franklin's Eqs. (2)-(6) are meaningless, for continually accelerated ships. They are not incorrect, they are meaningless, since there is no S' frame. (Thus Franklin's contention in section 1 of (2010) that 'it is only the rest frame length of an object that relates to strains and stresses on the object' is in the general case wrong.)

The correct distances between the *corresponding points* of ships B and C, in the case when the points are performing identical hyperbolic motions relative to S, as measured in instantaneous rest *frames* of B and C at the same instant of their proper time τ are given, e.g. in Peregoudov (2009) (cf. also Semay 2006, Redžić 2009). Note that one and the same inertial frame first becomes the instantaneous rest frame of B and only at a later instant (with respect to *that* frame) it becomes the instantaneous rest frame of C.

Eventually, Franklin's resolution of the Bell spaceship paradox 'as no paradox' is hard to fathom. It is of course true that Special Relativity allows no difference in any measurement of two equal lengths such as the distance between ships and the length of the thread between them. However, the two distances are not of the same sort in the following sense. Consider, for simplicity, the 'mild' variant of the problem when all transient effects have died out and a steady velocity of ships is reached in S. Then to the former distance the relativistic length reduction applies, whereas to the latter (the length of the thread) both the length reduction and a stretching above the natural relativistic FitzGerald–Lorentz contraction of the thread apply (under the proviso that the thread remained *unbroken* and under the proviso of course that Special Relativity is valid). While our Galilean instincts would expect the thread never breaks in S (why should it?), it must break at a sufficiently high speed if Special Relativity is valid, and this is the core of the paradox.

The above conclusion that 'a stretching above the natural FitzGerald–Lorentz contraction of the thread' applies in the mild variant of the problem is based on the tacit assumption that releasing the thread ends from ships in the final rest frame S' would lead to the thread's shrinking in S' to its initial rest length (the length it had in S before accelerations started), or in other words that the thread is perfectly elastic. Without that simplifying assumption the analysis of Dewan–Beran–Bell's problem becomes tricky in S. Thus, without that simplifying assumption Bell's resolution of the paradox in S is oversimplified. However, at a sufficiently high speed the thread would certainly break regardless of its elasticity since, according to Special Relativity, its length in S' would tend to infinity when $v \to c$. The same conclusion is reached in the S frame, taking into account that the thread's natural (FitzGerald–Lorentz contracted) length when it is in uniform motion would tend to zero when $v \to c$. (The appearance of Redžić (2008a) stimulated heated discussions on some internet forums and several published (Peregoudov 2009, Redžić 2009, Podosenov et al. 2010) and unpublished papers on the topic. An anonymous Russian author, a philosopher by profession, remarked that 'it would not be a big harm if a philosopher added into the barrel of professional physicists' honey a teaspoon of philosophical tar'. The above comment is prompted by the anonymous author's remark that Special Relativity *alone* does not imply the thread would break due to 'the artificial prevention of the natural contraction'.)

3.6. Rigid body motion in Special Relativity

The last section of Franklin (2010) entitled 'Rigid body motion in special relativity' begins by pointing out that in the motion described by Bell, the acceleration of each spaceship is the same at equal times in S. The author then contends 'this also corresponds to each [spaceship] having the same acceleration a' in their [mutual] instantaneous rest system [...] if their rest system acceleration is constant in time.' However, as was noted above, there is no common instantaneous rest frame for both ships; instead, ships' accelerations are constant in their respective instantaneous rest frames. In the same way, Franklin's next argument that 'from the preceding paragraph we see that keeping lengths constant in the rest system requires different rest frame accelerations for different parts of a rigid body' is inconclusive: there is no *mutual* instantaneous rest frame in Dewan-Beran-Bell's problem.

It should be stressed that Franklin's subsequent analysis of the motion of ships in the case of their constant but different rest frame accelerations, so as to keep the distance between ships constant in their mutual rest system, is exact and instructive. There is only one terminological point where I disagree with the author. Namely, his phrase 'rigid body motion', which reflects the concept originally introduced by Born (1909), should be replaced by a more appropriate term 'rigidly moving body' (cf. Pauli 1958, pp. 130-132, Miller 1981). Recently, expounding his 2010 argument, Franklin (2013) has attempted to clarify the terminology by introducing instead a 'relativistic rigid body'.

3.7. Linking electrical current and the pole in a barn paradox

In a recent paper, McGlynn and van Kampen (2008) contend that the phenomenon of alterations in charge densities in a current-carrying wire as measured by different inertial observers 'perfectly demonstrates "the pole in a barn" paradox'. However, this is wrong; the two phenomena exemplify two distinct aspects of relativistic length contraction, namely, the length reduction and the FitzGerald–Lorentz contraction, respectively. Since the confusion appears to be a recurrent point in various contexts (cf. Zapolsky 1988, Cavalleri and Tonni 2000), and taking into account that it is closely connected with our preceding considerations, it is perhaps worthwhile to briefly discuss McGlynn and van Kampen's contention.

The standard textbook derivations of the magnetic force that acts on a moving charge q via Special Relativity consider the case of an infinite straight wire at rest in the 'laboratory' frame (French 1968, Feynman et al. 1975, Purcell 1985). The wire is modelled as consisting of two superposed lines of charge: one moving (that of free electrons moving at drift speed v_d) and the other, which has an equal but opposite charge density, at rest (that of fixed positive ions). Thus, the wire is taken to be electrically neutral in the laboratory frame (S), which implies that the distance between adjacent ions equals the (mean) distance between adjacent electrons in S. Then by applying the relevant relativistic length reduction formulae (*mutatis mutandis* in Eq. (1)) to those distances, the corresponding charge densities in the rest frame of the moving charge q are found. Eventually, following the well-known relativistic path, making (tacitly) use of the happy circumstance that the Lorentz force is a *pure* relativistic force (Rindler 1991, Jefimenko 1996, Redžić et al. 2011), the desired result for the magnetic force is obtained. Note that in the above scene-setting no contractions are involved in the S frame. The same scene was used by McGlynn and van Kampen (2008), except for the fact that the authors confined their attention to a *segment* of the wire in S, which is irrelevant for the present discussion. (Note also that no contractions are involved (the thread apart) in the *distance* between the corresponding points of ships in Dewan–Beran–Bell's problem *in the S frame*.)

The situation is different in the pole in a barn problem (Dewan 1963, cf. also Rindler 1991). Namely, a pole vaulter (who, according to Dewan's original formulation, lives in 'Tompkins' Wonderland' where the speed of light is low) must speed up his pole from rest in a rest length–preserving way, so that the FitzGerald–Lorentz contraction formula (3) applies. In this case obviously there is contraction in the S frame (which is now the barn frame). Thus, there is a basic distinction between the two phenomena described above, contrary to McGlynn and van Kampen's claim.

In more detail, the FitzGerald–Lorentz contraction (and thus also the corresponding preparatory stage in which the pole acquires its motion relative to the barn) is essential in the pole in a barn problem; the contraction makes it possible that the pole in motion enters (momentarily) the barn (while this was impossible when the pole was at rest with respect to the barn). Consequently, a change of the pole with respect to the (inertial) barn frame, and thus the active aspect of length contraction, is essential in the problem.

We have another story in the phenomenon of alterations in charge densities in a current carrying wire as measured by *different inertial observers*. First, there is no contraction with respect to the wire (laboratory) frame, in the case of the steady state assumed by McGlynn and van Kampen (2008), Zapolsky (1988), French (1968), Feynman et al. (1975), Purcell (1985), cf. also Redžić (2012). Second, what is essential for the phenomenon is *the assumed steady state* (the wire carrying the steady current is electrically neutral in the wire frame); the preparatory stage (starting from electrically neutral wire with no current) is irrelevant for the phenomenon. Third, the alterations in charge densities are found using the relativistic length reduction formula, Eq. (1). Therefore, the passive aspect of length contraction is exemplified in the phenomenon.

Acknowledgements – I am grateful to Brian Coleman, Giuliano Boella, Vladimir Hnizdo, Jerrold Franklin, Dmitry Peregoudov, Paul van Kampen and Jaykov Foukzon for stimulating and cordial correspondence. I am also grateful to the anonymous referee for constructive comments. My work is supported by the Ministry of Science and Education of the Republic of Serbia, project No. 171028.

REFERENCES

Bell, J. S.: 1976, *Prog. Sci. Cult.*, **1**, (2), 1 (reprinted in Bell, J. S.: 1987, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, pp. 67–80).
Born, M.: 1909, *Ann. Phys., Lpz.*, **30**, 1.
Born, M.: 1963, *Nature*, **197**, 1287.
Born, M.: 1965, Einstein's Theory of Relativity, Dover, New York.
Brown, H. R.: 2005, Physical Relativity Spacetime Structure from a Dynamical Perspective, Clarendon, Oxford.
Brush, S. G.: 1967, *Isis*, **54**, 230.
Cavalleri, G. and Tonni, E.: 2000, *Phys. Rev. A*, **61**, 026101.
Cornwell, D. T.: 2005, *Europhys. Lett.*, **71**, 699.
Dewan, E. M.: 1963, *Am. J. Phys.*, **31**, 383.
Dewan, E. and Beran, M.: 1959, *Am. J. Phys.*, **27** 517.
Dieks, D.: 1984, *Zeitschrift für allgemeine Wissenschaftstheorie*, **15**, 330.
Dingle, H.: 1962, *Nature*, **195**, 985.
Einstein, A.: 1905, *Ann. Phys., Lpz.*, **17**, 891.
Evett, A. A. and Wangsness, R. K.: 1960, *Am. J. Phys.*, **28**, 566.
Feynman, R. P., Leighton, R. B. and Sands, M.: 1975, The Feynman Lectures on Physics, Vol 2, Addison–Wesley, Reading, MA, Sec 13–6.
FitzGerald, G. F.: 1889, *Science*, **13**, 390.
Franklin, J.: 2010, *Eur. J. Phys.*, **31**, 291.
Franklin, J.: 2013, *Found. Phys.*, **43**, 1489.
French, A. P.: 1968, Special Relativity, Nelson, London.
Gamow, G.: 1961, *Proc. Natl. Acad. Sci.*, **47**, 728.
Jefimenko, O. D.: 1996, *Am. J. Phys.*, **64**, 618.
Kraus, U.: 2008, *Eur. J. Phys.*, **29**, 1.
Lorentz, H. A.: 1892, *Versl. Kon. Akad. Wetensch* **1**, 74 (reprinted in translation: Lorentz, H. A.: 1937, Collected Papers, Vol 4, Nijhoff, The Hague, pp. 219–223.)
Lorentz, H. A., Einstein, A., Minkowski, H. and Weyl, H.: 1952, The Principle of Relativity, Dover, New York.
McGlynn, E. and van Kampen, P.: 2008, *Eur. J. Phys.*, **29** N63.
Mermin, N. D.: 2003, *Am. J. Phys.*, **71**, 296.
Miller, A. I.: 1981, Albert Einstein's Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905-1911), Addison–Wesley, Reading, MA.
Mirabelli, A.: 1985, *Am. J. Phys.*, **53**, 493.
Panofsky, W. K. H. and Phillips, M.: 1955, Classical Electricity and Magnetism, Addison–Wesley, Cambridge, MA.
Pauli, W.: 1958, Theory of Relativity, Pergamon, London.
Peregoudov, D. V.: 2009, *Eur. J. Phys.*, **30**, L3.
Podosenov, S. A., Foukzon, J., and Potapov, A. A.: 2010, *Grav. Cosmol.*, **16**, 307.
Purcell, E. M.: 1985, Electricity and Magnetism, McGraw-Hill, New York.
Redžić, D. V.: 2005, *Eur. J. Phys.*, **26**, 991.
Redžić, D. V.: 2006a, Recurrent Topics in Special Relativity: Seven Essays on the Electrodynamics of Moving Bodies, authorial edition, Belgrade.
Redžić, D. V.: 2006b, *Eur. J. Phys.*, **27**, 147.
Redžić, D. V.: 2008a, *Eur. J. Phys.*, **29**, N11.
Redžić, D. V.: 2008b, *Eur. J. Phys.*, **29**, 191.
Redžić, D. V.: 2009, *Eur. J. Phys.*, **30**, L7.
Redžić, D. V., Davidović, D. M. and Redžić, M. D.: 2011, *J. Electro. Waves Appl.*, **25**, 1146.
Redžić, D. V.: 2012, *Eur. J. Phys.*, **33**, 513.
Resnick, R.: 1968, Introduction to Special Relativity, Wiley, New York.
Rindler, W,: 1991, Introduction to Special Relativity, Clarendon, Oxford.
Rosser, W. G. V.: 1964, An Introduction to the Theory of Relativity, Butterworths, London.
Schrödinger, E.: 1977, What is Life & Mind and Matter, Cambridge University Press, Cambridge.
Semay, C.: 2006, *Eur. J. Phys.*, **27**, 1157.
Styer, D. F.: 2007, *Am. J. Phys.*, **75**, 805.
Terrell, J.: 1959, *Phys. Rev.*, **116**, 1041.
Torretti, R.: 2006, http://philsci-archive.pitt.edu/2875/
Zapolsky, H. S.: 1988, *Am. J. Phys.*, **56**, 1137.

APPENDIX

Assume a rod of unit length at rest in S, lying along the x-axis, taking up the segment between the origin and the point $x = 1$m. The adequate picture of the rod (one-dimensional in space) is a strip of the x, ct-plane, bounded by the ct-axis ($x = 0$) and the line $x = 1$m parallel to it. It is the strip as a manifold of world points which has *objective reality.* At various instants of the S-time, the rod is represented by cross-sections of the strip parallel to the x-axis. In the S' frame, however, the same rod is represented by cross-sections of the strip parallel to the corresponding x'-axis, at various instants of the S'-time; the length of the rod is $\sqrt{1 - v^2/c^2}\text{m}'$, as measured in S'. (Recall that the lesser S'-length is a *longer* line segment than the S-length on the corresponding Minkowski diagram, due to the well-known properties of space calibration hyperbola $x^2 - c^2t^2 = 1$.) Thus to one and the same objective reality (the strip) correspond various *physical realities* (cross-sections of the strip parallel to the corresponding spatial axes), being the world-maps of the *same* rod in various reference frames. In this sense, each inertial frame has its own physical reality.

Assume now that the velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ is imparted to the rod so that it moves uniformly along its length (the x-axis) with respect to S, and assume also that the acceleration was a rest length-preserving one. (This assumption is contained in Born's 'principle of the physical identity of the units of measure'.) In this case, the corresponding objective reality of the rod is depicted by a strip of the x, ct-plane inclining to the ct-axis, bounded by the ct'-axis ($x' = 0$), and the line $x' = 1\text{m}'$ parallel to it. Cross-sections of the inclined strip parallel to the x-axis are physical reality for the S-observer, their length being of course $\sqrt{1 - v^2/c^2}$m, whereas cross-sections parallel to the x'-axis are physical reality for the S'-observer, their length being 1m$'$ (cf. footnote 8 of the *Contraction*).

It is clear that there is a change in the object due to acceleration with respect to the S-frame (or, equivalently, due to deceleration with respect to the S'-frame): objective reality (the strip) has changed; this is so, of course, for all inertial observers. (However, despite the physical change has happened, the object is still one and the same object in the sense that it is still a bound configuration consisting of the same material points.)

In the above argument, Born's term 'physical reality' is replaced by 'objective reality'; on the other hand, I used 'physical reality' of an inertial observer as a synonym for Rindler's world-map (cf. Rindler 1991, and also the *Contraction*). Despite appearances, my term 'objective reality' does not necessarily imply a reality which would be independent of the realm of our perceptions. Note also that my argument is in accord with that presented by Minkowski in his famous address 'Space and Time' more than a hundred years ago (Lorentz et al. 1952, pp. 74-91).

ПРОДУЖЕНА АГОНИЈА РЕЛАТИВИСТИЧКЕ ДУЖИНЕ

D. V. Redžić

Faculty of Physics, University of Belgrade
PO Box 44, 11000 Beograd, Serbia

E–mail: *redzic@ff.bg.ac.rs*

УДК 52–334.2
Оригинални научни рад

У овом есеју покушали смо да коригујемо скорашње збуњујуће интерпретације неких фундаменталних релативистичких концепата и резултата. Базирајући аргументе на закључцима недавно публикованог рада (Redžić 2008b), анализирали смо погрешне концепције које су рекурентан садржај литературе посвећене настави релативности попут: нема промене објекта у Специјалној Релативности, илузорни карактер релативистичке контракције дужине, напони и релативне деформације индуковани Лоренцовом контракцијом, и сродне теме. Навели смо неколико примера замки свакодневног језика које вребају у Специјалној Релативности. У циљу уклањања могуће терминолошке и концептуалне збрке, увели смо дистинкцију између релативистичке редукције дужине и релативистичке Фицџералд–Лоренцове контракције, које одговарају пасивној и активној интерпретацији контракције дужине, респективно; указали смо да оба аспекта имају фундаменталан динамички садржај. Као илустрацију наших разматрања, кратко смо дискутовали Дјуан–Беран–Белов парадокс космичког брода и парадокс 'мотке у штали'.