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SEMI-ANALYTICAL FORMULAS FOR THE FUNDAMENTAL PARAMETERS OF GALACTIC EARLY B SUPERGIANTS

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SUMMARY: The publication of new tables of calibration of some fundamental parameters of Galactic B0-B5 supergiants in the two classes $I_{\rm a}$ and $I_{\rm b}$ allows to particularize the eight parameters conjecture that model five fundamental parameters. The numerical expressions for visual magnitude, radius, mass, luminosity and surface gravity are derived for supergiants in the range of temperature between 29700 K and 15200 K. The availability of accurate tables of calibration allows us to estimate the efficiency of the derived formulas in reproducing the observed values. The average efficiency of the new formulas, expressed in percent, is 94 for the visual magnitude, 81 for the mass, 96 for the radius, 99 for the logarithm of the luminosity and 97 for the logarithm of the surface gravity.

Key words. Stars: fundamental parameters – Hertzsprung-Russell (HR) and C-M – Stars: general

1. INTRODUCTION

A recent study on the supergiants of spectral type B0-B5, see Searle et al. (2008), provided a fine grid of the fundamental parameters such as the temperature, luminosity, radius, surface gravity and mass. In this paper we first review the four fundamental parameters as modeled by the eight parameters conjecture and we add to the list the surface gravity, see Section 2. Section 3 contains the numerical expressions for five fundamental parameters as well as the efficiency of such an evaluation.

2. THE FUNDAMENTAL FORMULAS

We briefly review the formulas that characterize the visual magnitude, M_V, the mass, $\mathcal{M}$, the radius, R, and the luminosity, L, of the stars for each MK class as function of the intrinsic, unreddened color index $(B-V)_0$, see Zaninetti (2008) for details. The first equation models the visual magnitude, M_V

$$
\begin{aligned}
M_V = & -2.5\,a_{\rm LM} - 2.5\,b_{\rm LM}\,a_{\rm MT} - \\
& 2.5\,b_{\rm LM}\,b_{\rm MT}\log_{10}(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}}) \\
& -K_{\rm BC} + 10\log_{10}(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}}) + \\
& \frac{T_{\rm BC}}{T_{\rm BV}}\left[(B-V)_0 - K_{\rm BV}\right] + M_{{\rm bol},\odot} \quad . \qquad (1)
\end{aligned}
$$

The second equation connects the mass of the star, $\mathcal{M}$, with $(B-V)_0$

$$\log_{10}(\frac{\mathcal{M}}{\mathcal{M}_\odot}) = a_{\rm MT} +$$
$$b_{\rm MT} \ln\left(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}}\right)(\ln(10))^{-1} \quad , \quad (2)$$

where $\mathcal{M}_\odot$ is the solar mass. The third equation relates the radius R, with $(B-V)_0$

$$\log_{10}(\frac{R}{R\odot}) =$$
$$1/2a_{\rm LM} + 1/2b_{\rm LM}a_{\rm MT} + 2\frac{\ln(T_\odot)}{\ln(10)} +$$
$$+1/2b_{\rm LM}b_{\rm MT}\ln\left(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}}\right)(\ln(10))^{-1}$$
$$-2\ln\left(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}}\right)(\ln(10))^{-1} \quad , \quad (3)$$

where $R\odot$ is the solar radius. The fourth equation connects the luminosity of a star, L, with $(B-V)_0$

$$\log_{10}(\frac{L}{L\odot}) = a_{\rm LM} + b_{\rm LM}a_{\rm MT}$$
$$+b_{\rm LM}\left(b_{\rm MT}\ln\left(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}}\right)\frac{1}{\ln(10)}\right) \quad , \quad (4)$$

where $L\odot$ is the solar luminosity. The eight numerical parameters compared above are reported in Table 1 along with the physical or empirical formula that relates them.

Table 1. Synoptic table of the eight coefficients. *BC* stands for the bolometric correction

Coefficient	adopted relationship
$K_{\rm BV}$,$T_{\rm BV}$	$(B-V)_0 = K_{\rm BV} + T_{\rm BV}/T$
$K_{\rm BC}$,$T_{\rm BC}$	$BC = -\frac{T_{\rm BC}}{T} - 10\ \log\ T + K_{\rm BC}$
$a_{\rm LM}$, $b_{\rm LM}$	$\log(L/L_\odot) = a_{\rm LM} + b_{\rm LM}\log(\mathcal{M}/\mathcal{M}_\odot)$
$a_{\rm MT}$, $b_{\rm MT}$	$\log(\mathcal{M}/\mathcal{M}_\odot) = a_{\rm MT} + b_{\rm MT}\log(T/T_\odot)$

A fifth fundamental parameter is the surface gravity, g, that is defined as

$$g = G\frac{\mathcal{M}}{R^2} \quad , \quad (5)$$

where $\mathcal{M}$ is the mass of the body, R its radius and G is the Newtonian gravitational constant $G = 6.6742 \times 10^{-11}\frac{\rm m^3}{\rm kgs^2}$ (Mohr and Taylor 2005).

By adopting $R_\odot = 6.95508 \cdot 10^8$ m and $\mathcal{M} = 1.989 \cdot 10^{30}$ kg, (see Cox 2000), we obtain the following expression for the logarithm of the surface gravity

$$\log_{10}(g[{\rm cm/s^2}]) = -10.60 + 0.4342\times$$
$$\times\ln(e^{2.3a_{\rm MT} - 2.3a_{\rm LM} - 2.3b_{\rm LM}\,a_{\rm MT}}$$
$$\times(\frac{T_{\rm BV}}{(B-V)_0 - K_{\rm BV}})^{b_{\rm MT} - b_{\rm LM}\,b_{\rm MT} + 4}) \quad . \quad (6)$$

3. APPLICATION TO THE SUPERGIANTS

The eight parameters conjecture may represent an acceptable fit of five fundamental parameters of the stars once the calibration data are available in the considered MK class, in our case B0-B5 supergiants. The Table 5 in Searle et al. (2008) provides the calibration of $\log_{10}(\frac{L}{L\odot})$, $\log_{10}(\frac{R}{R\odot})$, $\frac{\mathcal{M}}{\mathcal{M}_\odot}$ and $\log_{10}(g[{\rm cm/s^2}])$ as function of the temperature T in the range $15200 < T < 29700$. Table 2 reports the eight parameters as well as the source where the calibrated data are taken from.

The fundamental parameters of the stars are parametrized according to the MK class to which they belong, and to the intrinsic, unreddened color index $(B-V)_0$ or the temperature as derived, for example, from spectroscopic arguments. The conversion between temperature and $(B-V)_0$ is obtained through the following two formulas

Table 2. The adopted coefficients for B0-B5 supergiants.

Coefficient	Ia	Ib	source of data
$K_{\rm BV}$	-0.3961	-0.3961	Table 15.7 in Cox (2000)
$T_{\rm BV}$[K]	4011.7	4011.7	Table 15.7 in Cox (2000)
$K_{\rm BC}$	42.87	42.87	Table 15.7 in Cox (2000)
$T_{\rm BC}$[K]	31573.8	31573.8	Table 15.7 in Cox (2000)
$a_{\rm LM}$	4.667	4.092	Table 5 in Searle et al. (2008)
$b_{\rm LM}$	0.6050	0.92054	Table 5 in Searle et al. (2008)
$a_{\rm MT}$	-3.713	-6.0246	Table 5 in Searle et al. (2008)
$b_{\rm MT}$	1.1674	1.7213	Table 5 in Searle et al. (2008)

$$(B-V)_0 = -0.3961 + \frac{4011.7}{T} \tag{7}$$
$$15200\ \mathrm{K} < T < 30000\ \mathrm{K} \quad ,$$

$$T = \frac{4011.7}{(B-V)_0 + 0.3961} \tag{8}$$
$$-0.25 < (B-V)_0 < -0.14 \quad .$$

From a numerical point of view the visual magnitude, M_{V}, is given by

$$M_V = -41.07 + 3.576\ \ln(4012.0\ ((B-V)_0 + 0.3961)^{-1}) + 7.870\,(B-V)_0 \tag{9}$$
$$\text{supergiants Ia}\ -0.25 < (B-V)_0 < -0.14 \quad ,$$

and

$$M_V = -31.38 + 2.623\ \ln(4012.0\ ((B-V)_0 + 0.3961)^{-1}) + 7.870\,(B-V)_0 \tag{10}$$
$$\text{supergiants Ib}\ -0.25 < (B-V)_0 < -0.14 \quad .$$

Fig. 1. shows the theoretical visual magnitude, M_V, for the two classes considered here, as well as the observational data extracted from Table 3 of Searle et al. (2008).

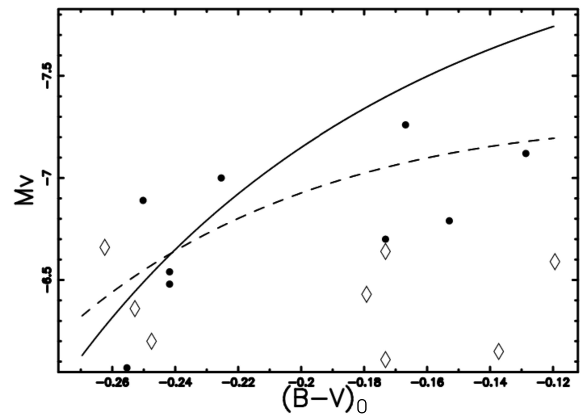

Fig. 1. *Theoretical visual magnitude,* M_{V}*, versus* $(B-V)_0$ *for Galactic early B supergiants: Ia (solid line) and Ib (dotted line). The observed values of magnitude extracted from Table 3 of Searle et al. (2008) are also shown: Ia (full dots) and Ib (empty diamonds).*

The mass $\frac{\mathcal{M}}{\mathcal{M}_\odot}$ can be expessed as

$$\frac{\mathcal{M}}{\mathcal{M}_\odot} = 10.0^{-3.713+0.5070\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \tag{11}$$
$$\text{supergiants Ia}\ -0.25 < (B-V)_0 < -0.14 \quad ,$$

and

$$\frac{\mathcal{M}}{\mathcal{M}_\odot} = 10.0^{-6.025+0.7476\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \tag{12}$$
$$\text{supergiants Ib}\ -0.25 < (B-V)_0 < -0.14 \quad .$$

Fig. 2. shows the theoretically determined mass, $\frac{\mathcal{M}}{\mathcal{M}_\odot}$, for the two classes considered here, as well as the observational data extracted from Table 4 of Searle et al. (2008).

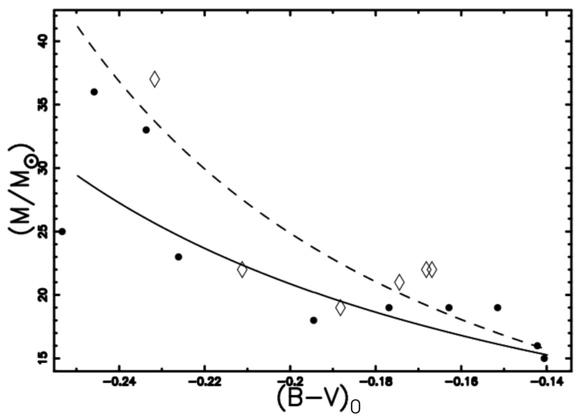

Fig. 2. *Theoretical mass,* $\frac{\mathcal{M}}{\mathcal{M}_\odot}$*, versus* $(B-V)_0$ *for Galactic early B supergiants: Ia (solid line) and Ib (dotted line). The observed values of mass extracted from Table 4 of Searle et al. (2008) are also shown: Ia (full dots) and Ib (empty diamonds).*

The radius, $\frac{R}{R_\odot}$, is given by expression

$$\frac{R}{R_\odot} = 10.0^{8.733-0.7151\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \tag{13}$$
$$\text{supergiants Ia}\ -0.25 < (B-V)_0 < -0.14 \quad ,$$

and

$$\frac{R}{R_\odot} = 10.0^{6.795-0.5245\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \tag{14}$$
$$\text{supergiants Ib}\ -0.25 < (B-V)_0 < -0.14 \quad .$$

Fig. 3. shows the theoretical radius, $\frac{R}{R_\odot}$, for the two classes considered here, as well as the observational data extracted from Table 4 of Searle et al. (2008).

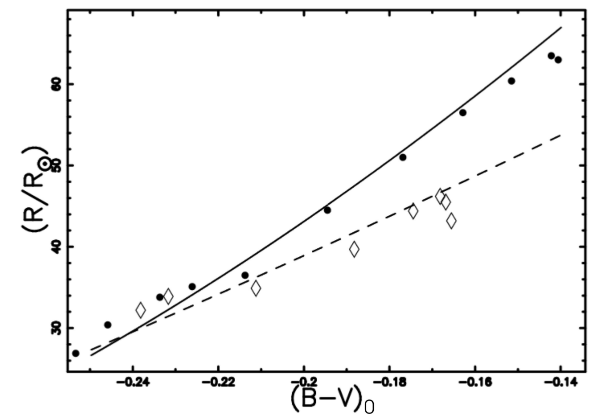

Fig. 3. *Theoretical radius,* $\frac{R}{R_\odot}$*, versus* $(B-V)_0$ *for Galactic early B supergiants: Ia (solid line) and Ib (dotted line). The observed values of radius extracted from Table 4 of Searle et al. (2008) are also shown Ia (full dots) and Ib (empty diamonds).*

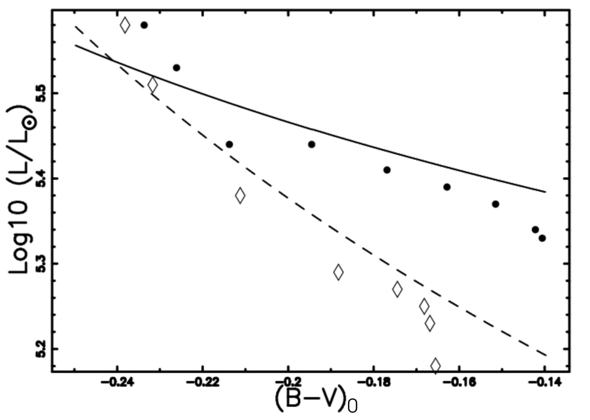

Fig. 4. *Theoretical logarithm of the luminosity,* $\log_{10}(\frac{L}{L\odot})$*, versus* $(B-V)_0$ *for Galactic early B supergiants: Ia (solid line) and Ib (dotted line). The observed values of luminosity extracted from Table 4 of Searle et al.(2008) are also shown: Ia (full dots) and Ib (empty diamonds).*

The logarithm of the luminosity, $\log_{10}(\frac{L}{L\odot})$, is given by

$$\log_{10}(\frac{L}{L\odot}) = 2.42 + 0.3\ln\left(4012\ ((B-V)_0+0.39)^{-1}\right) \quad (15)$$

$$\text{supergiants Ia} \quad -0.25 < (B-V)_0 < -0.14 \quad ,$$

and

$$\log_{10}(\frac{L}{L\odot}) = -1.45 + 0.68\ln\left(4012\,((B-V)_0+0.39)^{-1}\right) \quad (16)$$

$$\text{supergiants Ib} \quad -0.25 < (B-V)_0 < -0.14 \quad .$$

Fig. 4. shows the theoretically determined logarithm of the luminosity, $\log_{10}(\frac{L}{L\odot})$, for the two classes considered here, as well as the observational data extracted from Table 4 of Searle et al. (2008).

The logarithm of the surface gravity, $\log_{10} g$, is given by

$$\log_{10}(g[\mathrm{cm/s^2}]) = -10.61 + 0.4343 \times \ln\left(3.118\ \left(((B-V)_0+0.3961)^{-1}\right)^{1.167}\right) - 0.8686\ \ln\left(0.00001891\ \left(((B-V)_0 + 0.3961)^{-1}\right)^{-1.646}\right) \quad (17)$$

$$\text{supergiants Ia when} \quad -0.25 < (B-V)_0 < -0.14,$$

and

$$\log_{10}(g[\mathrm{cm/s^2}]) = -10.61 + 0.4343 \times \ln\left(1.506\ \left(((B-V)_0+0.3961)^{-1}\right)^{1.721}\right) - 0.8686\ \ln\left(0.000008342\ \left(((B-V)_0 + 0.3961)^{-1}\right)^{-1.2077}\right) \quad (18)$$

$$\text{supergiants Ib when} \quad -0.25 < (B-V)_0 < -0.14.$$

Table 3. Efficiency in deriving the fundamental parameters for B0-B5 supergiants

	Ia			Ib		
Fundamental Parameter	$\epsilon_{\rm min}$ (%)	$\bar{\epsilon}$ (%)	$\epsilon_{\rm max}$ (%)	$\epsilon_{\rm min}$ (%)	$\bar{\epsilon}$ (%)	$\epsilon_{\rm max}$ (%)
Visual magnitude	88.8	94.5	98.7	83.5	91.4	97.5
Mass	10.5	81.2	97.7	55.2	81.4	95.9
Radius	91.5	95.9	98.5	90.4	94.8	98.9
Logarithm Luminosity	98.7	99.2	99.6	98.3	99.1	99.6
Logarithm surface gravity	90.9	96.8	98.9	95.9	98.2	99.5

Fig. 5. shows the theoretically determined logarithm of the surface gravity for the two classes considered here, as well as the observational points extracted from Table 4 of Searle et al. (2008).

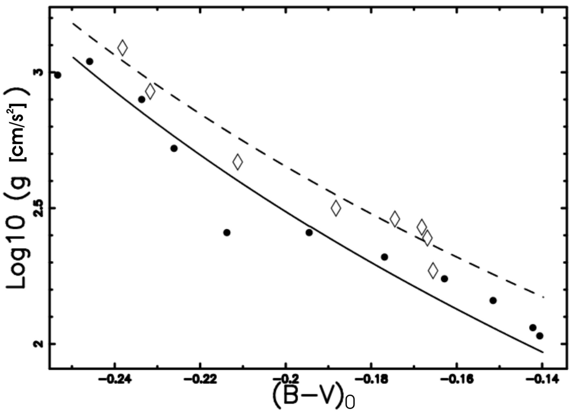

Fig. 5. *Theoretical logarithm of the surface gravity,* $\log_{10}(g[\mathrm{cm/s}^2])$*, versus* $(B-V)_0$ *for Galactic early B supergiants: Ia (solid line) and Ib (dotted line). The observed values of surface gravity extracted from Table 4 of Searle et al. (2008) are also shown: Ia (full dots) and Ib (empty diamonds).*

From the practical point of view, ϵ, a measure of the reliability of our results expressed in percents can also be introduced:

$$\epsilon = (1 - \frac{|(F_{\mathrm{obs}} - F_{\mathrm{num}})|}{F_{\mathrm{obs}}}) \cdot 100 \,, \qquad (19)$$

where F_{obs} is one of the fundamental parameters as given by the astronomical observations, and F_{num} the analogous fundamental parameter as given by our numerical relationships. The minimum, average and maximum efficiency in reproducing the observed fundamental parameters (respectively ϵ_{min}, $\bar{\epsilon}$ and ϵ_{max}), as given by formula (19), are reported in Table 3.

4. CONCLUSIONS

The eight parameters conjecture which allows to model five fundamental parameters of the stars is connected with the availability of calibration data on mass and luminosity as a function of the temperature or the intrinsic, unreddened color index $(B-V)_0$. Tables 3 and 4 in Searle et al. (2008) offer both the splitting of early supergiants in two classes and a good coverage of the fundamental parameters in the range $15200\ K < T < 30000\ K$. The five fundamental parameters analyzed here can be presented as functions of $(B-V)_0$ and Table 3 reports the efficiency of the model implemented here. As an example, the two relationships (10) and (11) pertaining to the visual magnitude have an average accuracy of 0.34 mag in Ia and 0.5 mag in Ib. This paper also presents the logarithm of the surface gravity, $\log_{10}(g[\mathrm{cm/s}^2])$, as a function $(B-V)_0$, see formulas (6), (18), (19) and Fig. 5. This allows to cast doubt to the fits of T that contain $\log_{10}(g[\mathrm{cm/s}^2])$ as a parameter, see for example Sekiguchi and Fukugita (2000), Kovtyukh et al. (2000), because the reverse is true: $\log_{10} g$ is a function of T or its observational counterpart $(B-V)_0$.

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ПОЛУ-АНАЛИТИЧКЕ ФОРМУЛЕ ЗА ОСНОВНЕ ПАРАМЕТРЕ ГАЛАКТИЧКИХ РАНИХ В СУПЕРЏИНОВА

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Оригинални научни рад

Дају се нове вредности осам калибрисаних коефицијената помоћу којих се изводе релације за апсолутну визуелну магнитуду, радијус, масу, луминозност и површинску гравитацију звезда у функцији ефективне температуре. Нумерички изрази ових параметара се односе на галактичке B0 – B5 суперџинове класе сјаја I_{a}, I_{b} и температуре између 15200 K и 28700 K. Тачност изведених формула у односу на посматране вредности параметара, у процентима је 94 за магнитуду, 81 за масу, 96 за радијус, 99 за логаритам сјаја и 97 за логаритам површинске гравитације.