

LUMINOSITY VARIATION IN THE EXTENDED  
ONE-ZONE RR LYRAE MODEL

D. Pricopi

*Astronomical Institute of the Romanian Academy  
Str. Cutitul de Argint 5, RO-040558 Bucharest, Romania*

(Received: March 15, 2005; Accepted: April 13, 2005)

**SUMMARY:** The Stellingwerf one-zone stellar model is extended by assuming, a slow and uniform rotation that leads to a very small oblateness of the star. The matter in the core-surrounding shell is supposed to consist of a mixture of ideal gas and radiation. This one-zone stellar pulsation model is proposed as a tool to investigate the factors affecting luminosity variations of pulsating stars. Linear and non-linear analyses of the resulting equations are described. The results are in very good agreement with the observed RR Lyrae light curves.

**Key words.** Stars: variables: RR Lyr – Stars: interiors

## 1. INTRODUCTION

The one-zone model was first introduced by Baker (1966) to simplify the problem of radial stellar pulsation. The assumption of small amplitude motion of a thin shell eliminates the spatial derivatives and the mathematical form becomes a simple cubic equation. This analytic approach has shed much light upon local destabilizing mechanisms and has been used to probe more complicated phenomena such as non-radial pulsation (Ishizuka 1967, Zahn 1968) and convection in variable stars (Unno 1967, Okamoto and Unno 1967, Gough 1967). Usher and Whitney (1968) have considered this model in the limit of a very thick shell (core radius  $R_c = 0$ ).

The non-linear one-zone model is represented mathematically as a nonlinear third-order set of ordinary differential equations with time as the independent variable. Rudd and Rosenberg (1970) presented a model in which the non-adiabaticity is assumed, obtaining a remarkable agreement with observations. The nonlinear one-zone model was concisely formulated and investigated in the linear and nonlinear

cases by Stellingwerf (1972). Taking into account the luminosity variation at the base of the shell, he obtained a very realistic light variation.

In the present paper, Stellingwerf's nonlinear one-zone model is extended by considering a slow and uniform rotation that leads to a very small oblateness of the star and that the matter in the core-surrounding shell consists of a mixture of ideal gas and radiation. Von Zeipel's theorem indicates the incompatibility of uniform rotation and generation of energy by nuclear reactions. However, the resulting meridional currents are very slow (Sweet 1950) so that, in this respect, uniform rotation remains an adequate work hypothesis. This model is not intended to be a substitute for finely zoned nonlinear calculations. The results are qualitative and we are content to seek out and explain only the simplest features in terms of basic physical processes. In Section 2 we resort to the well-known equations of stellar structure (Kippenhahn and Weigert 1991, Lungu 1982) and the equations for our model are written down. Section 3 deals with the physical input of the model. The linear results are presented in Section 4. The condition of phase lag of  $\pi/2$  (maximum luminosity after minimum radius) is obtained in terms of phys-

ical parameters. In Section 5 nonlinear results are presented. The conclusions of this paper are summarized in Section 6.

## 2. BASIC EQUATIONS

The equations of stellar structure are (Kippenhahn and Weigert 1991, Lungu 1982):

1. the motion equation:

$$\frac{\partial^2 r}{\partial t^2} = -\frac{Gm}{r^2} - \frac{3Gma^2\lambda}{5r^4}(1 - 3\cos^2\theta) + \omega^2 r \sin^2\theta - 4\pi r^2 \frac{\partial P}{\partial m}, \quad (1)$$

2. the continuity equation:

$$dm = 4\pi r^2(1 - \lambda)\rho dr, \quad (2)$$

3. the energy equation:

$$\frac{\partial l}{\partial m} = -c_V \frac{\partial T}{\partial t} + \frac{\delta}{\alpha} \frac{P}{\rho^2} \frac{\partial \rho}{\partial t}, \quad (3)$$

4. and the radiative energy transport equation in the diffusion approximation:

$$l = [4\pi r^2(1 - \lambda)]^2 \frac{4\sigma}{3\kappa} \frac{\partial T^4}{\partial m}, \quad (4)$$

where  $\omega$  is the (small) angular velocity,  $\lambda$  denotes the oblateness,  $a$  stands for the semimajor axis of the ellipsoid, and  $\theta$  is the polar angle. The other notations are usual.

Like in Stellingwerf (1972), we introduce the following relations referring to the core-surrounding shell:

$$\frac{\partial P}{\partial m} = -\frac{P}{m_s}, \quad \frac{\partial l}{\partial m} = \frac{L - L_i}{m_s}, \quad \frac{\partial T^4}{\partial m} = -\frac{T^4}{m_s}, \quad (5)$$

where  $P$ ,  $L$ ,  $T$  stand for the pressure, radiative energy flux, and temperature in the shell, respectively,  $L_i$  is the luminosity at the base of the shell and  $m_s$  denotes the shell mass. Let  $M$  be the stellar mass,  $R$  be the stellar radius and  $R_c$  the rigid core radius. The equations (1), (3) and (4) become respectively:

$$\frac{d^2 R}{dt^2} = -\frac{GM}{R^2} - \frac{3GMa^2\lambda}{5R^4}(1 - 3\cos^2\theta) + \omega^2 R \sin^2\theta + 4\pi R^2 \frac{P}{m_s}, \quad (6)$$

$$\frac{L - L_i}{m_s} = -c_V \frac{\partial T}{\partial t} + \frac{\delta}{\alpha} \frac{P}{\rho^2} \frac{\partial \rho}{\partial t}, \quad (7)$$

$$L = -\frac{64\pi^2(1 - \lambda)^2 \sigma R^4}{3\kappa} \frac{T^4}{m_s}. \quad (8)$$

From definition,  $\lambda = (a - b)/(a + b)$ , with  $b$  denotes the semiminor axis of the ellipsoid, so that we can write

$$a = \frac{R}{\sqrt{1 + \frac{4\lambda \cos^2\theta}{(1+\lambda)^2}}}. \quad (9)$$

Taking into account the expression (9), Eq. (6) becomes

$$\frac{d^2 R}{dt^2} = -\frac{GM}{R^2} \chi + \omega^2 R \sin^2\theta + 4\pi R^2 \frac{P}{m_s}, \quad (10)$$

where

$$\chi = 1 + \frac{3}{5} \frac{\lambda(1 - 3\cos^2\theta)}{1 + \frac{4\lambda \cos^2\theta}{(1+\lambda)^2}}. \quad (11)$$

For a static star ( $\omega = 0$ ), we have  $\lambda = 0$  and, from (11),  $\chi = 1$ .

The hydrostatic equilibrium state implies

$$4\pi R_0^2 \frac{P_0}{m_s} = \frac{GM}{R_0^2} \chi - \omega^2 R_0 \sin^2\theta, \quad (12)$$

where the subscript "0" corresponds to the equilibrium model.

Following Stellingwerf (1972), we denote

$$X = \frac{R}{R_0}. \quad (13)$$

The geometry is introduced via the function  $m = m(X)$  such that (Rudd and Rosenberg 1970)

$$\frac{\rho}{\rho_0} = X^{-m}, \quad (14)$$

where

$$m(X) = \frac{\ln\left(\frac{X^3 - \eta^3}{1 - \eta^3}\right)}{\ln X}, \quad (15)$$

with  $\eta = R_c/R_0$ . The equilibrium value of  $m$  is  $m_0 = 3/(1 - \eta^3)$ .

The non-adiabatic effects are contained in the function  $h$  defined by

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^{\Gamma_1} h \quad (16)$$

where  $\Gamma_1 = (\partial \ln P / \partial \ln \rho)_{ad}$ . With these definitions, Eq. (10) becomes

$$\frac{d^2 X}{dt^2} = \xi(hX^{-q} - X^{-2}) - \zeta(hX^{-q} - X) \quad (17)$$

where  $\xi = GM\chi/R_0^3$ ,  $\zeta = \omega^2 \sin^2\theta$ , and  $q = m\Gamma_1 - 2$ .

Regarding the properties of the stellar matter, we consider the following formulae for the equation of state and the opacity law, respectively:

$$\rho = \rho_k P^\alpha T^{-\delta}, \quad (18)$$

$$\kappa = \kappa_k \rho^n T^{-s}, \quad (19)$$

where  $\rho_k$  and  $\kappa_k$  are constants.

Using (14) and (16), we write

$$\frac{P}{P_0} = X^{-m\Gamma_1} h. \quad (20)$$

From (14), (18) and (20) we obtain

$$\frac{T}{T_0} = X^{-m\frac{\gamma-1}{\delta}} h^{\frac{\gamma}{\delta}}, \quad (21)$$

where  $\gamma = c_P/c_V = \alpha\Gamma_1$ . For the opacity we write, using (14), (19) and (21):

$$\frac{\kappa}{\kappa_0} = X^{-m[n-s\frac{\gamma-1}{\delta}]} h^{-s\frac{\gamma}{\delta}}. \quad (22)$$

From (8), (21) and (22), we obtain for luminosity:

$$\frac{L}{L_0} = X^{4+m[n-(4+s)\frac{\gamma-1}{\delta}]} h^{(4+s)\frac{\gamma}{\delta}}, \quad (23)$$

and the variation of the luminosity at the base of the shell is supposed to be

$$\frac{L_i}{L_0} = X^{-u}, \quad (24)$$

where  $u$  is a parameter that ranges from 0 to 20 (Stellingwerf and Donohoe 1987).

Using these relations, Eq. (7) becomes

$$\begin{aligned} \frac{dh}{dt} = & -\varepsilon \frac{\delta}{\alpha} X^{m\frac{\gamma-1}{\delta}} h^{1-\frac{\gamma}{\delta}} \times \\ & \times \left( X^{4+m[n-(4+s)\frac{\gamma-1}{\delta}]} h^{(4+s)\frac{\gamma}{\delta}} - X^{-u} \right) - \\ & - 3\frac{\gamma-1}{\alpha} X^2 h (X^3 - \eta^3)^{-1} \times \\ & \times \left( X^{m[1-\Gamma_1+\frac{\gamma-1}{\delta}]} h^{1-\frac{\gamma}{\delta}} - 1 \right) \frac{dX}{dt}, \quad (25) \end{aligned}$$

where  $\varepsilon = L_0/m_s c_V T_0 = L_0/E_s$  ( $E_s$  is the internal energy of the shell). Equations (17) and (25) constitute our final set of relations for the unknown quantities  $X$  and  $h$ .

### 3. PHYSICAL INPUT

For our model, we take  $M = 0.5M_\odot$ ,  $R_0 = 3.41 \times 10^{11} \text{ cm}$ ,  $s = 3$ ,  $\varepsilon = 10^{-4}$ . The shell thickness is chosen to comprise the outer 10 - 15% of the stellar radius. Also,  $n$  is determined by the periodicity condition. Angular velocity is tend to be  $\omega = (2\pi/2.6) \times 10^{-6}$  (period of the rotation  $\approx 30$  days) and oblateness is  $\lambda = 5 \times 10^{-7}$ . The pressure is  $P = P_{\text{gas}} + P_{\text{rad}}$ . Following Kippenhahn and Weigert (1991), let  $\beta = P_{\text{gas}}/P$ . It follows  $\alpha = 1/\beta$ ,  $\delta = (4-3\beta)/\beta$ ,  $\nabla_{\text{ad}} = [1 + (1-\beta)(4+\beta)/\beta^2]/[5/2 +$

$4(1-\beta)(4+\beta)/\beta^2]$  and  $\Gamma_1 = 1/(\alpha - \delta\nabla_{\text{ad}})$ . For  $\beta = 1$  ( pure gas) we have  $\Gamma_1 = 5/3$ , and if  $\beta = 0$  (pure radiation) we have  $\Gamma_1 = 4/3$ .

### 4. LINEAR RESULTS

We assume small amplitude motion and put  $x = X - 1$ ,  $h' = h - 1$ . The linearized form of Eqs. (17) and (25) read:

$$\frac{d^2 x}{dt^2} = [\xi(2-q) + \zeta(1+q)]x + (\xi - \zeta)h', \quad (26)$$

$$\frac{dh'}{dt} = -\frac{\delta}{\alpha}\varepsilon[(b+u)x + (4+s)h'], \quad (27)$$

where  $b = 4 + m_0[n - (4+s)(\gamma-1)/\delta]$ . As usual, we assume a time variation  $e^{i\sigma t}$  for all quantities. From Eqs. (26) and (27) we obtain:

$$h' = \frac{\sigma^2 - [\xi(2-q) + \zeta(1+q)]}{(\xi - \zeta)} x, \quad (28)$$

$$h' = -\frac{\frac{\delta}{\alpha}\varepsilon(b+u)}{i\sigma + \varepsilon(4+s)} x, \quad (29)$$

These equations may be combined to yield

$$(i\sigma)^3 + A(i\sigma)^2 + B(i\sigma) + C = 0 \quad (30)$$

where  $A = \varepsilon(4+s)$ ,  $B = [\xi(2-q) + \zeta(1+q)]$  and  $C = \varepsilon(4+s)[\xi(2-q) + \zeta(1+q)] + (\delta/\alpha)\varepsilon(b+u)(\zeta - \xi)$ .

Variations in the exterior luminosity will be controled by the energy equation. We may define  $l = L/L_0 - 1$ , and combine Eqs. (23) and (29) to obtain

$$l = \frac{i\sigma b - Au}{i\sigma + A} x, \quad (31)$$

or

$$l = \frac{x}{|i\sigma|^2 + 2\text{Re}(i\sigma)A + A^2}. \quad (32)$$

$$[b|i\sigma|^2 + \text{Re}(i\sigma)A(b-u) - A^2u] + i[\text{Im}(i\sigma)A(b+u)].$$

$\text{Re}(i\sigma)$  and  $\text{Im}(i\sigma)$  are functions of  $n, s, u, \dots$  (see Eq. 30). To have strictly periodic pulsations we have to find such values for the parameters that  $\text{Re}(i\sigma) = 0$ . In that case, Eq. (32) becomes

$$l = \frac{[b|i\sigma|^2 - A^2u] + i[\text{Im}(i\sigma)A(b+u)]}{|i\sigma|^2 + A^2} x. \quad (33)$$

It is easy to show that the condition for the phase lag  $\phi = 90^\circ$  is

$$\begin{aligned} b\text{Im}(i\sigma)^2 &= A^2u, \\ \text{Im}(i\sigma) &> 0. \end{aligned} \quad (34)$$

From the first of the relation (34) we see that  $u = 0$  (no interior luminosity variation) implies  $b = 0$ , i.e. we have no vibrational instability. Thus, we may have  $\phi = 90^\circ$  only if we allow for interior luminosity variation.

We return to Eq. (30). Let  $y = i\sigma$  and  $z = y + A/3$ . Eq. (30) becomes:

$$z^3 - \frac{v}{3}z + \frac{w}{27} = 0, \quad (35)$$

where  $v = A^2 - 3B$ ,  $w = 2A^3 - 9AB + 27C$ . Let  $p = [(-w + (-4v^3 + w^2)^{1/2})/2]^{1/3}$ . The solutions of Eq. (30),  $y_i = z_i - A/3$  are:

$$y_1 = \frac{1}{3} \left( \frac{v}{p} + p - A \right), \quad (36)$$

$$y_{2,3} = -\frac{1}{6} \left[ (1 \pm i\sqrt{3}) \frac{v}{p} + (1 \mp i\sqrt{3})p + 2A \right]. \quad (37)$$

It is easy shown that  $\text{Im}(y_{2,3}) \neq 0$  if and only if  $-4v^3 + w^2 \geq 0$ . Also, it is seen that  $y_1 \in \mathbf{R}$  in any case. For  $-4v^3 + w^2 \geq 0$  we have:

$$\text{Re}(y_{2,3}) = -\frac{1}{6} \left( \frac{v}{p} + p + 2A \right), \quad (38)$$

$$\text{Im}(y_{2,3}) = -\frac{i\sqrt{3}}{6} \left( \pm \frac{v}{p} \mp p \right). \quad (39)$$

The condition for strictly periodic solutions ( $\text{Re}(y_{2,3}) = 0$ ) is

$$\frac{v}{p} + p + 2A = 0, \quad (40)$$

and they have the period:

$$\Pi = \frac{2\pi}{\frac{\sqrt{3}}{6} \left| \frac{v}{p} - p \right|}. \quad (41)$$

Because  $y_{2,3}$  are complex conjugate numbers, we may assume, without restricting the generality, that  $\text{Im}(y_2) > 0$ . The condition (34) for  $\phi = 90^\circ$  becomes:

$$\frac{b}{12} \left( \frac{v}{p} - p \right)^2 = A^2u, \quad (42)$$

and, using (40),

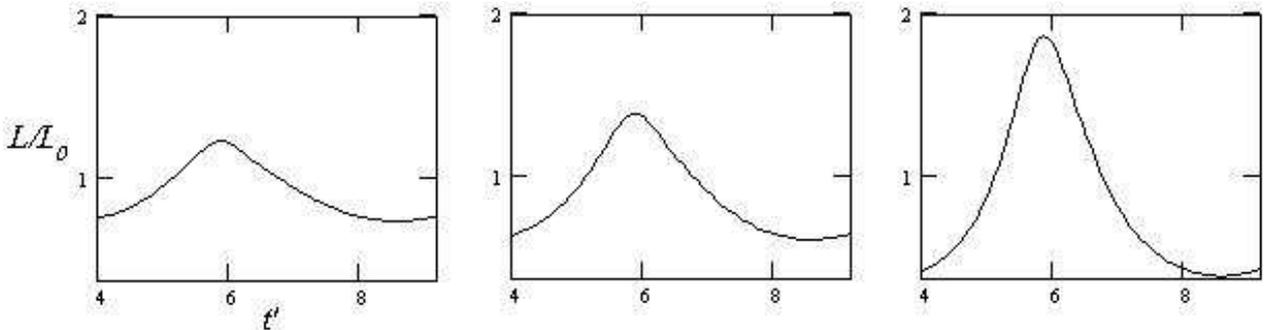
$$\frac{b}{3} (p + A)^2 = A^2u. \quad (43)$$

From (43) we have  $u > 0 \Leftrightarrow b > 0$  (vibrational instability).

## 5. NONLINEAR RESULTS

To facilitate nonlinear calculations, we use the time normalization  $t' = t/10^4$  and integrate Eqs. (17) and (25) using Mathcad software. The integrations were started at  $X = 1$ ,  $h = 1$  and  $dX/dt' = 10^{-5}$  (leading to a maximal radial velocity of few tens  $m/s$  for this model).

We begin by examining the effect of interior luminosity variation on the light curve taking  $u = 3$ , 5 and 10. In addition, we take  $\beta = 1$ ,  $\eta = 0.87$ ,  $\varepsilon = 10^{-4}$  and  $\omega = (2\pi/2.6) \times 10^{-6}$ . The resulting light curves are shown in Fig. 1.



**Fig. 1.** *The effect of interior luminosity variations.*

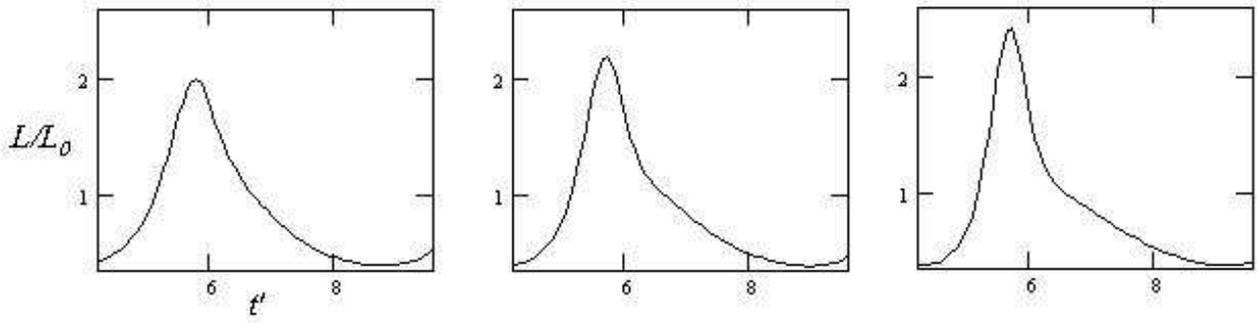


Fig. 2. The influence of radiation pressure.

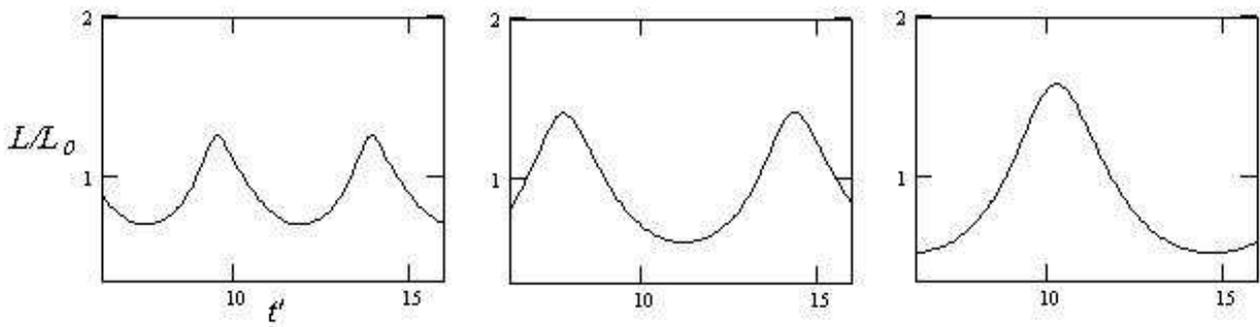


Fig. 3. The influence of the thickness of the shell.

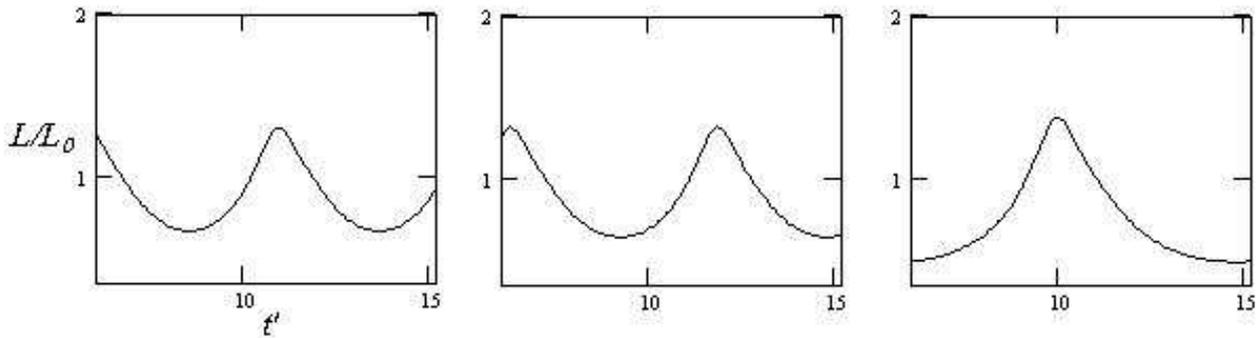


Fig. 4. The effect of the uniform rotation.

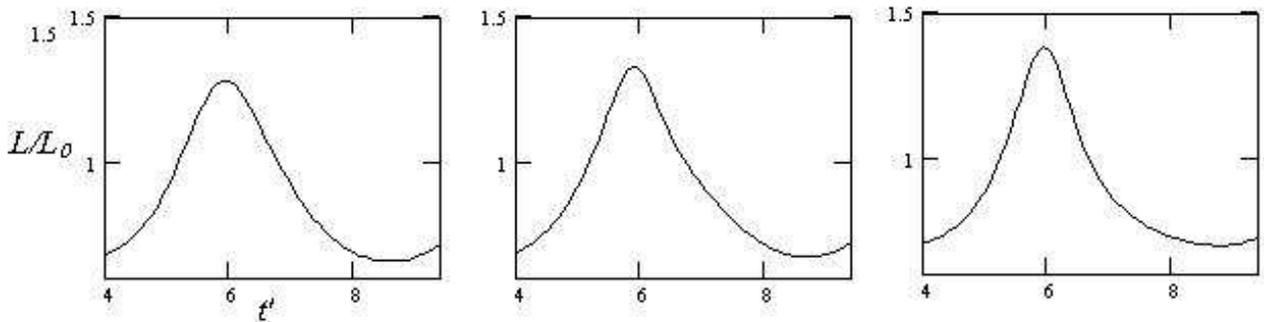


Fig. 5. The influence of  $\epsilon$ .

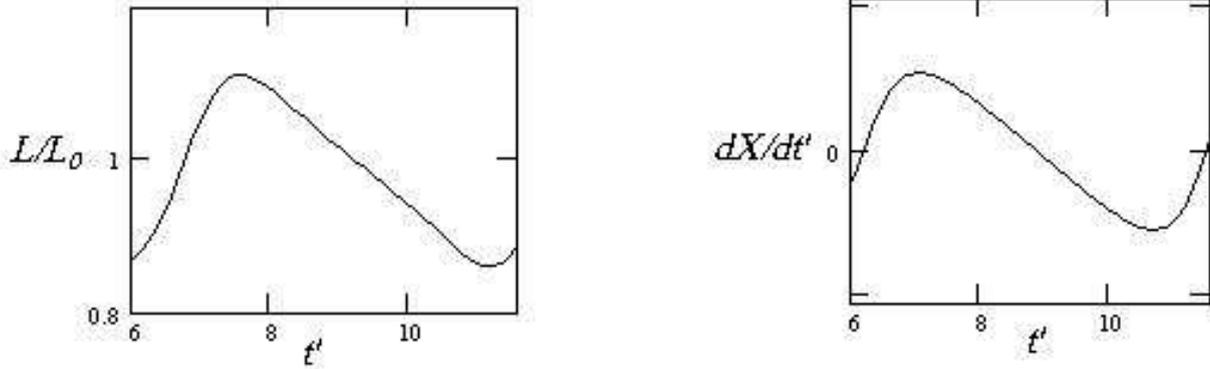


Fig. 6. The phase lag of about  $90^\circ$ .

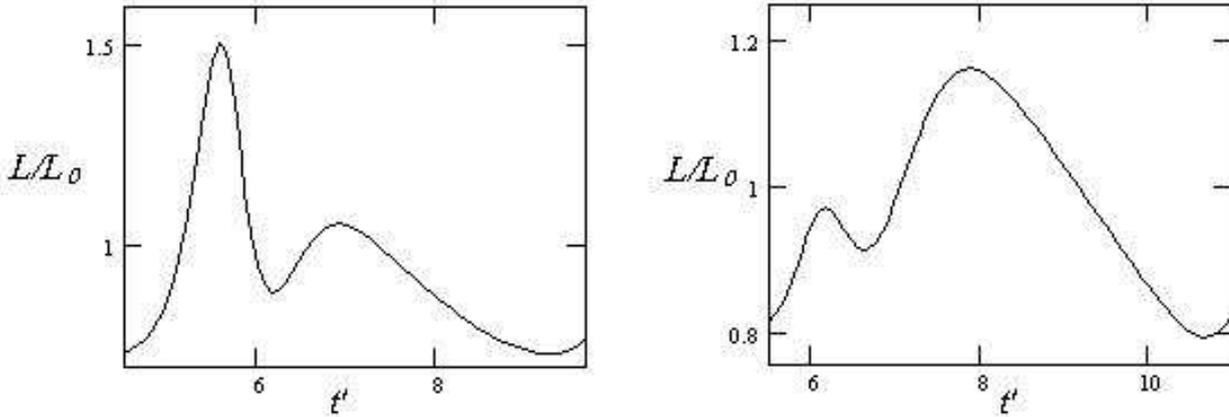


Fig. 7. The Blazhko effect.

The variations in magnitude are  $\Delta m \approx 0.557$  (typical of RR Lyrae of *c* type) for  $u = 3$ ,  $\Delta m \approx 0.893$  (typical of RR Lyrae of *b* type) for  $u = 5$  and  $\Delta m \approx 1.716$  (typical of RR Lyrae of *a* type) for  $u = 10$ . The period is found to be  $\Pi = 0.6$  days (typical of RR Lyrae type stars), same for all three values of  $u$ .

To investigate the effect of radiation, we take  $\beta = 0.99995$ ,  $\beta = 0.9999$  and  $\beta = 0.99985$ . In addition, we take  $u = 10$ ,  $\eta = 0.87$ ,  $\varepsilon = 10^{-4}$  and  $\omega = (2\pi/2.6) \times 10^{-6}$ . The resulting light curves are shown in Fig. 2.

While radiation pressure increases, the asymmetry becomes lower, and the shape of light curve resembles the observed light curves (we note (Ledoux and Walraven 1958) that the asymmetry is 0.4 - 0.5 for RR Lyrae of *c* type, 0.2 - 0.3 for RR Lyrae of *b* type and 0.1 - 0.2 for RR Lyrae of *a* type). Also, while radiation pressure is increasing, the amplitude is increasing:  $\Delta m \approx 1.774$  for  $\beta = 0.99995$ ,  $\Delta m \approx 1.898$  for  $\beta = 0.9999$ , and  $\Delta m \approx 2.05$  for

$\beta = 0.99985$ . The period is found to be  $\Pi = 0.625$  days, the same for all three values of  $\beta$ .

To investigate the effect of thickness of the shell, we take  $\eta = 0.7$ ,  $\eta = 0.8$  and  $\eta = 0.9$ . In addition, we take  $u = 4$ ,  $\beta = 1$ ,  $\varepsilon = 10^{-4}$  and  $\omega = (2\pi/2.6) \times 10^{-6}$ . The resulting light curves are shown in Fig. 3.

While the shell thickness increases, the period of pulsations increases too. The period is found to be  $\Pi = 0.532$  days for  $\eta = 0.9$ ,  $\Pi = 0.834$  days for  $\eta = 0.8$ ,  $\Pi = 1.134$  days for  $\eta = 0.7$ . Thus, for RR Lyrae type pulsating stars, we expect to have a shell thickness of about 10-15 of the stellar radius. This is confirmed by detailed calculations.

Now, let us investigate the effect of rotation. We take  $\omega = 0.1 \times (2\pi/2.6) \times 10^{-6}$ ,  $\omega = 5 \times (2\pi/2.6) \times 10^{-6}$  and  $\omega = 10 \times (2\pi/2.6) \times 10^{-6}$ . In addition, we take  $u = 4$ ,  $\beta = 1$ ,  $\varepsilon = 10^{-4}$  and  $\eta = 0.87$ . The resulting light curves are shown in Fig. 4.

While the rotation increases, the period of pulsations increases too. The period is found to be  $\Pi =$

0.613 days for  $\omega = 0.1 \times (2\pi/2.6) \times 10^{-6}$ ,  $\Pi = 0.671$  days for  $\omega = 5 \times (2\pi/2.6) \times 10^{-6}$ ,  $\Pi = 1.053$  days for  $\omega = 10 \times (2\pi/2.6) \times 10^{-6}$ . Thus, for RR Lyrae type pulsating stars, we expect to have a period of rotation of about a month. This is in good agreement with observations.

To investigate the effect of effective temperature, we take  $\varepsilon = 0.1 \times 10^{-4}$ ,  $\varepsilon = 0.5 \times 10^{-4}$  and  $\varepsilon = 5 \times 10^{-4}$ . In addition, we take  $u = 4$ ,  $\beta = 1$ ,  $\eta = 0.87$  and  $\omega = (2\pi/2.6) \times 10^{-6}$ . The resulting light curves are shown in Fig. 5.

While the ratio of luminosity to internal energy of the shell decreases, the amplitude of pulsations increases. This is in good agreement with the balance of energy: if the radiated energy decreases, the kinetic energy of the shell mass increases.

So for the pulsations are strictly periodic (the periodicity is maintained by  $\kappa$  - mechanism, keeping  $s = 3$  and finding appropriate values for  $n$ ), but phase lag  $\phi = 0$ . To obtain  $\phi = 90^\circ$ , we allow for a slow increase of the amplitude of pulsations. In Fig. 6, the light and velocity light curves are represented. We remark the very good agreement with the observed ones.

Consequently, for this model we cannot obtain exact the periodicity and the phase lag of about  $90^\circ$ . But, we can explain the Blazhko effect (the apparition of another peak in the light curve) by the increase of the radiation pressure.

In the first panel of Fig. 7, we take  $u = 2$ ,  $\beta = 0.9999$ ,  $\eta = 0.87$ ,  $\varepsilon = 10^{-4}$  and  $\omega = (2\pi/2.6) \times 10^{-6}$ , and in the second we take  $u = 10^{-3}$ ,  $\beta = 0.99998$ ,  $\eta = 0.86$ ,  $\varepsilon = 0.24 \times 10^{-4}$  and  $\omega = (2\pi/2.6) \times 10^{-6}$ .

## 6. CONCLUSION

Here we summarize the main results.

- (a) the increase of the interior luminosity variation leads to an increase of the amplitude of the light curve;
- (b) if the radiation pressure increases, the asymmetry decreases and the amplitude of the light curve increases;
- (c) if the shell thickness increase, the period of pulsations increases as well;

- (d) if the rotation increases, the period of pulsations also increases. For RR Lyrae type pulsating stars, we expect to have a period of rotation of about a month, in agreement with observations;
- (e) if the ratio of luminosity to internal energy of the shell decreases, the amplitude of pulsations increases;
- (f) for this model we cannot obtain exact the periodicity and the phase lag of about  $90^\circ$ ;
- (g) we can explain the Blazhko effect by the increase the radiation pressure in the shell.

Finally, we conclude that this simple model can explain (at least qualitatively) most of the features of the light curves of the RR Lyrae type pulsating stars.

## REFERENCES

- Baker, N.: 1966, Stellar Evolution, Eds. R. F. Stein and A. W. G. Cameron, Plenum Press, New York, p. 333
- Castor, J.I.: 1968, *Astrophys. J.*, **154**, 793.
- Castor, J.I.: 1971, *Astrophys. J.*, **166**, 109.
- Cox, J.P., Giuli: 1968, Principles of Stellar Structure, Gordon and Breach, New York.
- Cox, J.P., King, D.S., Stellingwerf, D.F.: 1972, *Astrophys. J.*, **171**, 93.
- Goldreich and Weber: 1980, *Astrophys. J.*, **238**, 991.
- Iben, I.Jr.: 1971, *Astrophys. J.*, **166**, 131.
- Kippenhahn, R., Weigert, A.: 1991, Stellar Structure and Evolution, Springer-Verlag, Berlin.
- Ledoux, P., Walraven, Th.: 1958, *Hbd. Phys.*, **50**, 353.
- Okamoto, I., Unno, W.: 1967, *Publ. Astr. Soc. Japan*, **19**, 154.
- Rudd, T.J., Rosenberg, R.M.: 1970, *Astron. Astrophys.*, **6**, 193.
- Stellingwerf, R.F.: 1972, *Astron. Astrophys.*, **21**, 91.
- Stellingwerf, R.F., Gautshy, A., Dickens, R.J.: 1987, *Astrophys. J.*, **313**, 75.
- Stellingwerf, R.F., Donohoe, M.: 1987, *Astrophys. J.*, **314**, 252.
- Sweet, P.A.: 1950, *Mon. Not. R. Astron. Soc.*, **110**, 548.
- Usher, P.D., Whitney, C.A.: 1968, *Astrophys. J.*, **154**, 203.

**ПРОМЕНЕ СЈАЈА У ПРОШИРЕНОМ ЈЕДНО-ЗОНСКОМ МОДЕЛУ RR LYRAE**

**D. Pricopi**

*Astronomical Institute of the Romanian Academy  
Str. Cutitul de Argint 5, RO-040558 Bucharest, Romania*

UDK 524.332 : 524.3-82  
*Оригинални научни рад*

Једно-зонски модел (Стелингверф 1972) за променљиве звезде RR Lyrae у овом раду је проширен, увођењем споре униформне ротације звезде која се састоји од смеше идеалног гаса и зрачења црног тела. Описана је линеарна и нелинеарна анализа изведених једначина. Нумерички резултати за изабра-

не вредности улазних параметара модела се веома добро слажу са посматраном кривом сјаја RR Lyrae. Овај модел се предлаже као алат за теоријско истраживање утицаја различитих фактора који подржавају промене сјаја пулсационих звезда.