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RADAR TIME DELAYS IN THE DYNAMIC THEORY OF GRAVITY

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SUMMARY: There is a new theory gravity called the dynamic theory, which is derived from thermodynamic principles in a five dimensional space, radar signals travelling times and delays are calculated for the major planets in the solar system, and compared to those of general relativity. This is done by using the usual four dimensional spherically symmetric space-time element of classical general relativistic gravity which has now been slightly modified by a negative inverse radial exponential term due to the dynamic theory of gravity potential.

Key words. Gravitation – Relativity – Methods: analytical – Techiques: radar astronomy

1. INTRODUCTION

There is a new theory called the Dynamic Theory of Gravity (DTG). It is derived from classical thermodynamics and requires that Einstein's postulate of the constancy of the speed of light holds (Williams 1997). Given the validity of the postulate, Einstein's theory of special relativity follows right away (Williams 2001). The dynamic theory of gravity (DTG) through Weyl's quantum principle also leads to a non-singular electrostatic potential of the form:

$$V(r) = -\frac{K}{r}e^{-\frac{\lambda}{r}}\,, \qquad (1)$$

where K is a constant and λ is a constant defined by the theory. The DTG describes physical phenomena in terms of five dimensions: space, time and mass (Williams 2001). By conservation of the fifth dimension we obtain equations which are identical to Einstein's field equations and describe the gravitational field. These equations are similar to those of general relativity and are given by:

$$K_o T^{\alpha\beta} = G^{\alpha\beta} = R^{\alpha\beta} - \frac{g^{\alpha\beta}}{2}R\,. \qquad (2)$$

Here $T^{\alpha\beta}$ is the surface energy-momentum tensor which may be found within the space tensor and is given by:

$$T^{\alpha\beta} = T^{\alpha\beta}_{sp} - \frac{1}{c^2}\left[F^{\alpha}_{4}F^{4\beta} - \frac{h^{\alpha\beta}}{2}F^{4\nu}F_{4\nu}\right]\,, \qquad (3)$$

and $T^{\mu\nu}_{sp}$ is the space energy-momentum tensor for matter under the influence of the gauge fields also given by Williams (2001):

$$T^{ij}_{sp} = \gamma u^i u^j + \frac{1}{c^2}\left[F^{i}_{k}F^{kj} + \frac{1}{4}a^{ij}F^{k\ell}F_{k\ell}\right]\,. \qquad (4)$$

This can further be written in terms of the surface metric in the following way:

$$T^{\alpha\beta}_{sp} = \gamma u^\alpha u^\beta + \frac{1}{c^2}\left[F^{\alpha}_{k}F^{k\beta} + F^{\alpha}_{4}F^{4\beta} + \frac{1}{4}\left(g^{\alpha\beta} - h^{\alpha\beta}\right)\left(F^{\mu\nu}F_{\mu\nu} + F^{4\nu}F_{4\nu}\right)\right] \qquad (5)$$

and:

$$u^4 = \frac{dy^4}{dt} \Rightarrow \frac{\partial y^4}{\partial t} + \bar{\nabla} \bullet \left(y^4 \, \bar{u} \right) = 0 \,. \qquad (6)$$

This is a statement required, by the conservation of the fifth dimension, and the surface indices ν, α, β. = 0,1,2,3 and space index i, j, k, l = 0,1,2,3,4, and

$$g_{\alpha\beta} = a_{ij} y^i_\alpha y^j_\alpha = a_{\alpha\beta} + h_{\alpha\beta} = a_{\alpha\beta} + 2a_{\alpha 4} y^4_\beta + a_{44} y^4_\alpha y^4_\beta \,,$$

where the surface field tensor will be given by:

$$F_{\alpha\beta} = F_{ij} y^i_\alpha y^j_\beta \text{ and } y^i_\alpha = \frac{\partial y^i}{\partial x^\alpha} = \delta^i_\alpha \qquad (7)$$

for $i = 0, 1, 2, 3$ and $y^4_\alpha = \frac{\partial y^4}{\partial x^\alpha}$.

$$F_{ij} = \begin{bmatrix} o & E_1 & E_2 & E_3 & V_o \\ -E_1 & o & B_3 & -B_2 & V_1 \\ -E_2 & -B_3 & o & B_1 & V_2 \\ -E_3 & B_2 & -B_1 & o & V_3 \\ -V_o & -V_1 & -V_2 & -V_3 & o \end{bmatrix} . \qquad (8)$$

It was shown by Weyl that the gauge fields may be derived from the gauge potentials and the components of the 5-dimensional field tensor F_{ij} given by the 5×5 matrix given by (8). Now the determination of the fifth dimension may be seen, for the only physically real property that could give Einstein's equations is the gravitating mass or it's equivalent mass, (Hunter et al. 1997). Finally the dynamic theory of gravity further argues that the gravitational field is a gauge field linked to the electromagnetic field in a five-dimensional manifold of space-time and mass, but, when conservation of mass is imposed, it may be described by the geometry of the four-dimensional hyper-surface of space-time, embedded into the five-dimensional manifold by the conservation of mass. The five-dimensional field tensor can only have one nonzero component V_0, which must be related to the gravitational field, and the fifth gauge potential must be related to the gravitational potential.

The theory makes its predictions for red-shifts by working in this five dimensional geometry of space, time, and mass, and determines the unit of action in the atomic states in a way that can be calculated with the help of quantum Poisson brackets when covariant differentiation is used (William 2001):

$$[x^\mu, p^\nu] \, \Phi = i\hbar g^{\nu q} \left\{ \delta_{\mu q} + \left| \Gamma^\mu_{s,q} \right| x^s \right\} \Phi \,. \qquad (9)$$

In (9) the vector curvature is contained in the Christoffel symbols of the second kind and the gauge function Φ is a multiplicative factor in the metric tensor $g^{\nu q}$, where the indices take the values: ν,q = 0,1,2,3,4. In the commutator, x^μ and p^ν are the space and momentum variables respectively, and finally $\delta_{\mu q}$ is the Kronecker delta. In DTG the momentum ascribed as a variable canonically conjugated to the mass is the rate at which mass may be converted into energy. The canonical momentum is defined as follows:

$$p_4 = m v_4 \qquad (10)$$

where the velocity in the fifth dimension, is given by:

$$v_4 = \frac{\dot{\gamma}}{\alpha_o} \,. \qquad (11)$$

Now $\dot{\gamma}$ is a time derivative, gamma having units of mass density (kg/m^3), α_o is a density gradient with units of kg/m^4. In the absence of curvature (8) becomes:

$$[x^\mu, p^\nu] \, \Phi = i\hbar \delta^{\nu q} \Phi \,. \qquad (12)$$

2. THE LINE ELEMENT OF THE DYNAMIC THEORY OF GRAVITY

In the DTG the metric is not different from that of general relativity except for an exponential term with an 1/r dependence, and λ is a constant determined by the theory. Therefore we can write the line element of dynamic gravity in the following way:

$$\begin{aligned} ds^2 &= c^2 \left(1 - \frac{2GM}{c^2 r} e^{-\lambda/r} \right) dt^2 - \\ & \left(1 - \frac{2GM}{c^2 r} e^{-\lambda/r} \right)^{-1} dr^2 - \\ & r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \,. \end{aligned} \qquad (13)$$

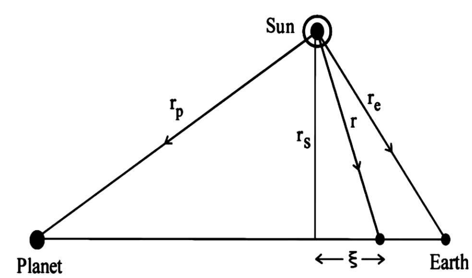

Fig. 1. *The relative position of a planet, with respect to the earth. A schematic diagram of the radar-ranging time delay experiment. Radar waves are sent from the Earth to a distant reflector, so that they pass close to the Sun. They are reflected as all electromagnetic radiation is. There is an excess time delay between sending and return above what would be expected were the signals propagating along straight lines in flat space-time. Time delay caused by the curvature of the space-time in the vicinity of the Sun is an important test of general relativity (Hartle 2003).*

3. RADAR DELAY IN THE DYNAMIC THEORY OF GRAVITY

With reference to Fig. 1 we can write that:

$$t_{\mathrm{dyn}_{(\mathrm{tot})}} = 2\left[\int_0^{\sqrt{r_e^2-r_s^2}} \frac{d\xi}{c'} + \int_0^{\sqrt{r_p^2-r_s^2}} \frac{d\xi}{c'}\right]. \quad (14)$$

If we now define $\xi = \sqrt{r^2 - r_s^2}$ we then have that $d\xi = \frac{rdr}{\sqrt{r^2-r_s^2}}$ and finally the round travel time becomes:

$$t_{\mathrm{dyn}_{(\mathrm{tot})}} = 2\left[\int_{r_s}^{r_e} \frac{dr}{\left[1 - \frac{2GMe^{-\lambda/r}}{rc^2}\right]\sqrt{1-\left(\frac{r_s}{r}\right)^2}} + \int_{r_s}^{r_P} \frac{dr}{\left[1 - \frac{2GMe^{-\lambda/r}}{rc^2}\right]\sqrt{1-\left(\frac{r_s}{r}\right)^2}}\right]. \quad (15)$$

Since λ, $r_S < r$ to first order approximation, the integral above takes the form:

$$t_{\mathrm{dyn}}(\mathrm{tot}) = \frac{2}{c}\left[\int_{r_S}^{r_e} \frac{\left[1 + \frac{2G}{rc^2} + \frac{2GM\lambda}{r^2c^2}\right]}{\sqrt{1-\left(\frac{r_s}{r}\right)^2}} dr + \int_{r_s}^{r_p} \frac{\left[1 + \frac{2GM}{rc^2} + \frac{2GM\lambda}{r^2c^2}\right]}{\sqrt{1-\left(\frac{r_s}{r}\right)^2}} dr\right]. \quad (16)$$

Since $r_s > 0$ and $r_e > r_s$ (15) and $\lambda = GM_{\mathrm{sun}}/c^2$, (15) can be written as:

$$t_{\mathrm{dyn}}(\mathrm{tot}) = \frac{2}{c}\left[\int_{r_S}^{r_e} \frac{\left[1 + \frac{2\lambda}{r} + \frac{2\lambda^2}{r^2}\right]}{\sqrt{1-\left(\frac{r_s}{r}\right)^2}} dr + \int_{r_s}^{r_p} \frac{\left[1 + \frac{2\lambda}{r} + \frac{2\lambda^2}{r^2}\right]}{\sqrt{1-\left(\frac{r_s}{r}\right)^2}} dr\right] \quad (17)$$

which now integrates to the following expression:

$$t_{\mathrm{dyn}}(\mathrm{tot}) = \frac{2}{c}\left[\sqrt{r^2 - r_s^2} - \frac{2\lambda^2}{r}\sin^{-1}\left(\frac{r_s}{r}\right) + 2\lambda\ \ln\left(r\left(1+\sqrt{1-\left(\frac{r_s}{r}\right)^2}\right)\right)\right]. \quad (18)$$

In (16) r_s is the distance of closest approach taken to be equal to the radius of the sun, r_e is the earth's orbital radius, r_p is the orbital radius of the planet, and λ is a parameter defined by the dynamic theory of gravity which has the value $\lambda = GM_{\mathrm{sun}}/c^2$. Substituting for the limits, expression (16) can be further simplified if we always remember that $r_s << r_e$, r_p to:

$$t_{\mathrm{dyn}}(\mathrm{tot}) = \frac{2}{c}\left[\sqrt{r_e^2 - r_s^2} + \sqrt{r_p^2 - r_s^2} + \frac{4\lambda^2}{r_s} - \frac{2\lambda^2}{r_e} - \frac{2\lambda^2}{r_p} + 2\lambda \ln\left(\frac{4r_e r_p}{r_s^2}\right)\right]. \quad (19)$$

The above equation results in a delay between classical signal propagation and that of dynamic gravity which is equal to:

$$\Delta t = t_{\mathrm{dyn}}(\mathrm{tot}) - t_{\mathrm{clas}}(\mathrm{tot}) = \frac{2}{c}\left[\frac{4\lambda^2}{r_s} - \frac{2\lambda^2}{r_e} - \frac{2\lambda^2}{r_p} + 2\lambda\ \ln\left(\frac{4r_e r_P}{r_s^2}\right)\right]. \quad (20)$$

Also the delay between general relativity and dynamic gravity takes the form:

$$\Delta t_{\mathrm{dyn}}(\mathrm{tot}) - t_{\mathrm{rel}}(\mathrm{tot}) = \frac{2}{c}\left[\frac{4\lambda^2}{r_s} - \frac{2\lambda^2}{r_e} - \frac{2\lambda^2}{r_p}\right]. \quad (21)$$

Next we will numerically evaluate equation (16) and we will compare it with that of general relativity:

$$t_{\mathrm{rel}}(\mathrm{tot}) = \frac{2}{c}\left[\int_{r_s}^{r_e} \frac{\left(1 + \frac{2\lambda}{r}\right)dr}{\sqrt{1-\left(\frac{r_s}{r}\right)^2}} + \int_{r_s}^{r_p} \frac{\left(1 + \frac{2\lambda}{r}\right)dr}{\sqrt{1-\left(\frac{r_s}{r}\right)^2}}\right] \quad (22)$$

which integrates into the expression:

$$t_{\mathrm{rel}}(\mathrm{tot}) = \frac{2}{c}\left[\sqrt{r_e^2 - r_s^2} + \sqrt{r_p^2 - r_s^2} + 2\lambda \ln\left(\frac{4r_e r_p}{r_s^2}\right)\right]. \quad (23)$$

4. CALCULATION OF RADAR TRAVELING TIMES

Our numerical calculations of the predicted total traveled times for the major planets in the solar system are shown in Table 1 and where in the third column the two digits in the bracket represents the only difference between the indicated travelling total times in the fifteen digital accuracy calculation. This difference, and for all practical purposes can be considered to be the same, between the two theories. All the planetary distances r_p are the planetary orbital radii and r_e is the orbital radius of the earth (Allen 2000). For this planetary configuration between the sun the earth and the planets we can easily see that $\mathrm{r}_s << r_p$, r_e.

Table 1.

Planetary Orbital Radii $\times 10^6$ Km	Classical Total Time (min)	Relativistic and Dynamic Total time (min)	Relativistic Delay (μsec)	Dynamic Delay (μsec)	Dynamic Delay (μsec)
Mercury 57.909175	23.071657 8730607	23.071661 3736106 (18)	210.03298 9764298	210.03306520 0644	209.996 5082
Venus 108.20893	28.664604 5366178	28.664608 2329327 (39)	221.77889 6458422	221.77897210 7128	221.742 4150
Mars 227.93664	41.977001 2778753	41.977005 2074730 (43)	235.77586 0526579	235.77593630 3704	235.739 3958
Jupiter 778.41202	103.18338 0980308	103.18338 5294488 (89)	258.85082 9682436	258.85090554 1638	258.814 3482
Saturn 1426.7274	175.26830 7725353	175.26831 2229253 (54)	270.23402 9226854	270.23410510 1500	270.197 5480

5. WHAT DISTANCE OF CLOSEST APPROACH MAKES DTG DELAYS ZERO?

Solving relations (20) and (21) for the distance of the closest approach r_s that will make the corresponding delays equal to zero, we obtain:

$$r_s = \frac{\lambda}{\text{ProductLog}\left[\pm\frac{\lambda e^{\frac{1}{2}\left(\frac{\lambda}{r_p}+\frac{\lambda}{r_e}\right)}}{2\sqrt{r_e r_p}}\right]} \approx \frac{\lambda}{\text{ProductLog}\left[\pm\frac{\lambda}{2\sqrt{r_e r_p}}\right]} \tag{24}$$

and also:

$$r_s = \frac{2r_e r_p}{(r_e + r_p)} \tag{25}$$

where ProductLog(z) function is the principal solution, of equations of the form $z = we^W$ also satisfying the following differential equation: $(dw/dz) = w/z(1+w)$ (Mathematica 4.0, 1999).

6. COMPARING DYNAMIC RELATIVISTIC AND CLASSICAL EFFECTS

The changes in the delay times between the dynamic and the classical propagation now become:

$$\frac{\Delta t_{\text{dyn}}}{t_{\text{dyn}}} = \frac{t_{\text{dyn}} - t_{\text{clas}}}{t_{\text{clas}}} \approx \frac{4\lambda^2}{r_s\,(r_e + r_p)}\left[1 + \frac{r_s}{2\lambda}\ln\left(\frac{4r_e r_p}{r_s^2}\right)\right], \tag{26}$$

and between the dynamic time and relativistic time we obtain:

$$\frac{\Delta t_{\text{dyn}}}{t_{\text{dyn}}} = \frac{t_{\text{dyn}} - t_{\text{rel}}}{t_{\text{rel}}} \approx \frac{4\lambda^2}{r_s\left[(r_e + r_p) + 2\lambda\,\ln\left(\frac{4r_e r_p}{r_s^2}\right)\right]}. \tag{27}$$

The Table 2 shows the magnitude of these quantities for the earth fixed at the position of its orbital distance and similarly the planets at their orbital radii which of course is not the most realistic position for radar measurements.

Table 2.

Planetary Orbital Radii $\times 10^6$Km	$\frac{\Delta t}{t} = \frac{t_{\mathrm{dyn}} - t_{\mathrm{rel}}}{t_{\mathrm{rel}}}$	$\frac{\Delta t}{t} = \frac{t_{\mathrm{dyn}} - t_{\mathrm{clas}}}{t_{\mathrm{clas}}}$
Mercury: 57.909175	10^{-14}	10^{-7}
Venus: 108.20893	10^{-14}	10^{-7}
Mars: 227.93664	10^{-14}	10^{-7}
Jupiter: 778.41202	10^{-14}	10^{-8}
Saturn: 1426.7274.	10^{-15}	10^{-8}

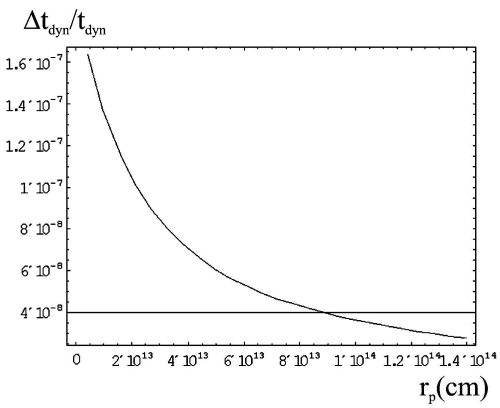

Fig. 2. $\Delta t_{dyn}/t_{dyn}$ *vs planetary distances at perihelion* r_p *(cm).*

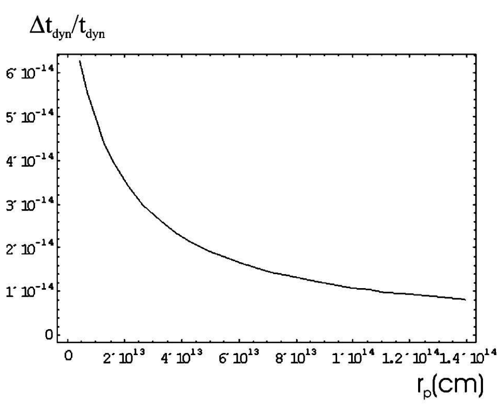

Fig. 3. $\Delta t_{dyn}/t_{dyn}$ *vs. planetary distances at perihelion* r_p *(cm).*

7. CONCLUSIONS

We have given a short introduction to the dynamic theory of gravity. Next an approximate first order calculation has been performed for obtaining the total traveling times of a radar signals in the neighbourhood of the sun transmitted from the earth. The planets considered are the ones indicated in the Table 1 and all of them were assumed to be away at their orbital radii distances. When actual measurements of this kind are carried out it could happen that this might not be the best planetary configuration. In any case this is a standard calculation that somebody can perform to test of a new gravitational theory. It is anticipated that the effects will be more evident if the planets are closer to the sun which is the main massive body affecting the signals gravitationally. It is also known that, at the position of a superior conjunction, the delay effects will become greater (Ohanian 1994).

From the numerical calculations of the total traveling times, it appears that there is not much of a significant difference, between dynamic gravity, general relativity but classical propagation differs being slightly smaller. To get an idea on the magnitude of this difference expressions describing the changes of the delay times between dynamic gravity and general relativity, as well as between dynamic gravity and classical propagation, have been derived and numerically evaluated for all the different planets and the results are compared.

The delay difference between dynamic gravity and general relativity is of the order of 10^{-14} for all examined planets except Saturn for which it is of the order of 10^{-15}. Next, the delay difference between dynamic gravity and classical propagation for all planets appears to be of the order of 10^{-7} except for Jupiter and Saturn for which it is of the order of 10^{-5}. Since dynamic gravity results in the same field equations as general relativity, it would not be unjustified to expect that delay effects on radio-signals would not differ much from those of general relativity, and any difference would be really small but not identical to that of general relativity. - With the help of the evolving present day and also future technology, such time differences might soon be accessible so that the validity of dynamic gravity as compared to general relativity might be finally understood and assessed.

REFERENCES

Allen: 2000, Astrophysical Quantities, 4^{th} Edition, AIR Press-Springer, p. 8.

Danby, J.M. A.:1989, Celestial Mechanics, Willman-Bell Inc., Second Edition, p. 6.

Hartle, James, B.: 2003, Gravity, An Introduction to Einstein's General Relativity, Addison Wesley, p. 213.

Hunter, G., Jeffers, S., Vigier, J.P.: 1997, Causality and Locality in Modern Physics, Kluwer Academic Publishers, York University, Canada, p. 261-268.

Ohanian, H.C. and Ruffini, R.: 1994, Gravitation and Spacetime, W.W. Norton and Company, p. 198.
Williams, P.E.: 1997, *Modern Physics Letters A 12*, **35**, 2725.
Williams, P.E.: 2001, *Apeiron*, **8**, No. 2, April, 2001, p. 84.
Williams, P.E.: 2001, *Entropy*, **3**, p. 76.
Williams, P.E. 2001, *Entropy*, **3**, p. 106.

КАШЊЕЊЕ РАДАРСКОГ СИГНАЛА У ДИНАМИЧКОЈ ТЕОРИЈИ ГРАВИТАЦИЈЕ

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UDK 52–16
Оригинални научни рад

У новој, алтернативној теорији, званој динамичка теорија гравитације, која је изведена из термодинамичких принципа у петодимензионом простору, времена путовања радарских сигнала и њихова одступања израчуната су за главне планете сунчевог система и упоређена са онима из опште релативности. Ово је урађено користећи четвородимензиони сферно симетрични елемент класичне релативности који је незнатно измењен негативним инверзним радијалним експоненцијалним чланом који потиче из динамичке теорије гравитације.