

THE CONVECTIVE COVER OF THE STELLAR MODEL HAVING THE CHEMICAL COMPOSITION: $X = 0.628$ AND $Z = 0.047$

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SUMMARY: In this paper the way the integrated equations describe the structure of the convective cover is presented. Also given are the numerical results of the convective cover considering null and not-null conditions at the surface of the star.

Key words. Stars: interiors – Stars: abundances – Methods: numerical

1. INTRODUCTION

A star model with radiative nucleus and a convective cover has been suggested. A reason for having chosen such a model is the following: we can notice granules on the surface of the photosphere, and there are turbulent movements within the photosphere, so we can speak of a convective equilibrium in its inferior part and of radiative equilibrium in its superior part. On one hand if we consider that the whole interior of the star is in convective equilibrium, the obtained values should be too high considering its surface temperature, and on the other hand if we consider that the whole interior is in radiative equilibrium, we should obtain negative values for the helium proportion. Among the models for the interior of the star we notice Schwarzschild's model and Sears' s evolutive model.

2. PRESENTATION OF THE PROBLEM

A model of the star is considered having the radiative nucleus and a convective cover. In Tatomir (2000) the way we have obtained the numerical results for the radiative nucleus is shown.

For the whole convective cover of the star, we

consider that the equation of the hydrostatic equilibrium, the equation of the mass distribution and the adiabatic equation are valid (see, e.g. Menzel et al. 1963, Aller and McLaughlin 1965, Cox and Giuli 1968).

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$P(r) = K \cdot \rho^\gamma(r) \quad (1)$$

or

$$P(r) = K_1 \cdot T(r)^{\frac{\gamma}{\gamma-1}}$$

with $\gamma = \frac{5}{3}$ and also the law of the perfect gas is considered to hold:

$$P(r) = \frac{k}{\mu \cdot H} \rho(r) \cdot T(r) \quad (2)$$

We use Schwarzschild's equations (Schwarzschild 1958):

$$P(r) = p \cdot \frac{GM^2}{4\pi R^4}$$

$$T(r) = t \frac{\mu H}{k} \cdot \frac{GM}{R} \quad (3)$$

$$M(r) = q \cdot M$$

$$r = R \cdot x$$

and we apply these equations to system (1), where x , q , t , p are adimensional variables. System (1) becomes:

$$\frac{dp}{dx} = -\frac{pq}{tx^2}$$

$$\frac{dq}{dx} = \frac{px^2}{t} \quad (4)$$

$$p = E \cdot t^{2.5}$$

or

$$\frac{dt}{dx} = -\frac{1}{2.5E} \cdot \frac{pq}{t^{2.5}x^2}$$

System (4) can be integrated using the null conditions for pressure and temperature at the surface of the star:

$$x = 1$$

$$t = p = 0 \quad (5)$$

$$q = 1$$

or using the not-null conditions at the surface of the star. Due to the fact that pressure is not well known at the base of the atmosphere of the star, we considered $T(r) = T_{ef} = 5785 \text{ K}$ and we did some integrations for (4) using different values for E .

The not-null conditions at the surface of the star are:

$$x = 1$$

$$q = 1 \quad (6)$$

$$t = \frac{1.3805 \times 6.96 \times 5.785}{1.672 \times 6.67 \times 1.99 \times 10^4} \times \frac{1}{\mu}$$

The mean molecular weight is given by:

$$\mu = \frac{4}{3 + 5X - Z} \quad (7)$$

In Tatomir (2000), there have been considered for the radiative nucleus, and also for the convective cover, the next two values:

$$X = 0.628 \quad (8)$$

$$Z = 0.047$$

representing the proportion of the hydrogen, and the abundance of the metals respectively.

We introduce three new parameters:

$$U = \frac{d \log M(r)}{d \log r}$$

$$V = -\frac{d \log P(r)}{d \log r} \quad (9)$$

$$(n + 1) = \frac{d \log P(r)}{d \log T(r)}$$

After making the calculations in (9), we obtain:

$$U = 4\pi r^3 \frac{\rho(r)}{M(r)} = \frac{px^3}{qt}$$

$$V = \frac{\rho(r)}{P(r)} \cdot \frac{GM(r)}{r} = \frac{q}{tx} \quad (10)$$

$$(n + 1)_{conv} = 2.5$$

3. NUMERICAL SOLUTION OF THE PROBLEM

Starting with the integration of the equations of the radiative nucleus (Tatomir 2000) from center, it stops when $(n + 1)_{rad} = 2.5$. In the point where the integration of the equation of the nucleus has stopped and where we fit the solution of the radiative nucleus with the one of the convective cover, we obtain the values in Tatomir (2000):

$$U_0 = 0.0067 \quad (11)$$

$$V_0 = 24.1433$$

3.1. The integration of the equation of the convective cover using null conditions at the surface of the star

We use for the system (4) the limit conditions (5). The system (4) has a non-determination of the type $\frac{0}{0}$ for $x = 1$. Using the development in Taylor series around the point $x = 1$, it has been obtained:

$$p(x) = \frac{E}{(2.5)^{2.5}} \cdot (1-x)^{2.5} + \dots$$

$$q(x) = 1 - \frac{E}{(2.5)^{2.5}} \cdot (1-x)^{2.5} + \dots \quad (12)$$

$$t(x) = \frac{1}{2.5}(1-x) + \frac{14E}{4+25E}(1-x)^2 + \dots$$

From (12) we have obtained the values of the parameters p , q , t in a point around $x = 1$. Next, the integration of the system (4) is made using the Runge-Kutta method (see, e.g. Moszynsky 1973, Tatomir 2000). We make integration choosing different values for the constant E .

In each point of integration x_i , we calculate $U(x_i)$ and $V(x_i)$. We integrate system the (4) until:

$$V(x_i) < V_0 \quad (13)$$

In the point x_i in which we apply (13). We test the condition:

$$|U(x_i) - U_0| < \varepsilon_1 \quad (14)$$

where $\varepsilon_1 = 10^{-5}$.

If the condition (14) is fulfilled, we consider the integration finished. If the condition (14) is not fulfilled, we continue the integration for another value of E , value which we denote E_1 .

$$E_1 = E + h_1$$

if $U_0 - U(x_i) > \varepsilon_1$, and

$$E_1 = E - h_1 \quad (15)$$

if $U_0 - U(x_i) < -\varepsilon_1$.

Initially, we consider $h_1 = 0.2$ and if the condition (14) is not satisfied, we start a new integration with $\frac{h_1}{2}$. The integration step is

$$h = 0.001044$$

and

$$x_i = 1 - i \cdot h \quad (16)$$

3.2. The integration of the equation of the convective cover using not-null conditions at the surface of the star

We integrate the system (4) using the conditions (6) for the surface of the star. The integration is performed by choosing different values for the parameter E , and considering the conditions (13)-(15). System (4) has no longer singularity in the point $x = 1$, which means that the integration starts directly with the Runge-Kutta method.

4. RESULTS AND CONCLUSIONS

If we consider the null conditions (5) for pressure and temperature at the surface of the star, we obtain:

$$E = 0.92$$

and in Table 1 are given the corresponding results for pressure P , the reduced mass q , temperature T and the density ρ , which correspond to the convective cover.

Table 1. The results for the null conditions at the surface of the star

x	P	q	T	ρ
1	0	1	0	0
0.999	$1.6085 \cdot 10^{-11}$	1	0.0125	$1.0204 \cdot 10^{-5}$
0.9948	$2.5725 \cdot 10^{-10}$	1	0.0379	$5.3976 \cdot 10^{-5}$
0.9844	$3.0912 \cdot 10^{-9}$	1	0.1024	$2.3995 \cdot 10^{-4}$
0.974	$1.0706 \cdot 10^{-8}$	1	0.1684	$5.0562 \cdot 10^{-4}$
0.9636	$2.483 \cdot 10^{-8}$	1	0.2357	$8.3759 \cdot 10^{-4}$
0.9532	$4.7109 \cdot 10^{-8}$	1	0.3045	$1.23 \cdot 10^{-3}$
0.9428	$7.92 \cdot 10^{-8}$	0.9999	0.3749	$1.6799 \cdot 10^{-3}$
0.9324	$1.2282 \cdot 10^{-7}$	0.9999	0.4468	$2.1858 \cdot 10^{-3}$
0.922	$1.7975 \cdot 10^{-7}$	0.9998	0.5203	$2.747 \cdot 10^{-3}$
0.9116	$2.5191 \cdot 10^{-7}$	0.9998	0.5955	$3.3636 \cdot 10^{-3}$
0.9012	$3.4133 \cdot 10^{-7}$	0.9997	0.6724	$4.036 \cdot 10^{-3}$
0.896	$3.9318 \cdot 10^{-7}$	0.9997	0.7116	$4.3934 \cdot 10^{-3}$

Table 2. The results for the not-null conditions at the surface of the star

x	P	q	T	ρ
1	$2.3746 \cdot 10^{-12}$	1	$5.785 \cdot 10^{-3}$	$3.2638 \cdot 10^{-6}$
0.999	$1.4467 \cdot 10^{-11}$	1	0.0119	$9.6516 \cdot 10^{-6}$
0.9898	$1.1338 \cdot 10^{-9}$	1	0.0682	$1.3215 \cdot 10^{-4}$
0.9796	$5.908 \cdot 10^{-9}$	1	0.132	$3.5582 \cdot 10^{-4}$
0.9694	$1.6104 \cdot 10^{-8}$	1	0.1972	$6.4942 \cdot 10^{-4}$
0.9592	$3.3313 \cdot 10^{-8}$	1	0.2637	$1.0045 \cdot 10^{-3}$
0.949	$5.9099 \cdot 10^{-8}$	0.9999	0.3317	$1.4168 \cdot 10^{-3}$
0.9388	$9.5053 \cdot 10^{-8}$	0.9999	0.4011	$1.8843 \cdot 10^{-3}$
0.9286	$1.4283 \cdot 10^{-7}$	0.9999	0.4721	$2.4058 \cdot 10^{-3}$
0.9184	$2.0418 \cdot 10^{-7}$	0.9998	0.5446	$2.9811 \cdot 10^{-3}$
0.9082	$2.8095 \cdot 10^{-7}$	0.9998	0.6187	$3.6103 \cdot 10^{-3}$
0.897	$3.8556 \cdot 10^{-7}$	0.9997	0.7023	$4.3653 \cdot 10^{-3}$

If we consider the not-null conditions (6) for pressure and temperature at the surface of the star, we obtain:

$$E = 0.9325$$

and in Table 2 there are the corresponding results for pressure, reduced mass, temperature and density.

In Tables 1 and 2, pressure P is in units of $10^{18} \text{ dyn} \cdot \text{cm}^{-2}$, the temperature T is in units of 10^6 K , the density ρ in $\text{g} \cdot \text{cm}^{-3}$, and q is the reduced mass.

We will compare the results that we have obtained in this paper with the ones from Tatomir (2000). In Table 3, the results are given that we have obtained in the fitting point x_i in Tatomir (2000). In Table 4 are given the values that correspond to the point x_i , using the null conditions at the surface of the star, and in Table 5 are the results using the not-null conditions at the surface of the star.

Table 3. The results for the nucleus from the previous paper of the author (Tatomir 2000)

x_i	P	q	T	ρ
0.897	0.3855E-6	0.9997	0.7023	0.0043

Table 4. The results of the interference of the nucleus with the cover for the null conditions

x_i	P	q	T	ρ
0.8956	0.4158E-6	0.9996	0.8110	0.0040

Table 5. The results of the interference of the nucleus with the cover for the not-null conditions

x_i	P	q	T	ρ
0.896	0.3931E-6	0.9997	0.7116	0.0043

By comparing the results which have been presented in Tables 3-5, we can conclude that the results that we have obtained in this paper agree well with the ones that have been obtained in Tatomir (2000) for the radiative nucleus. Another conclusion that can be drawn is that the utilization of the null conditions at the surface of the star is not restrictive and the corresponding numerical results are good.

The values of the constants that appear in this paper are:

$$G = 6.672 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$$

$$R = 6.96 \times 10^{10} \text{ cm}$$

$$H = 1.6725 \times 10^{-24} \text{ g}$$

$$M = 1.99 \times 10^{33} \text{ g}$$

$$k = 1.3805 \times 10^{-16} \text{ erg} \cdot \text{K}^{-1}$$

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КОНВЕКТИВНИ ОМОТАЧ У МОДЕЛУ ЗВЕЗДЕ ХЕМИЈСКОГ САСТАВА

$X = 0.628, Z = 0.047$

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Оригинални научни рад

Представља се модел конвективног омотача звезде задате масе и хемијског састава. Излаже се процедура решавања једначина структуре омотача и дају се нумерички резултати за случај нултих и фотосферских

граничних услова за температуру и притисак. Анализира се утицај ова два типа граничних услова на структуру омотача као и на карактеристике његове границе са радијативним језгром звезде.