Serb. Astron. J. № 157 (l998), 31 – 34

UDC 524.382
Original scientific paper

ON THE DISTRIBUTION OF VISUAL DOUBLE STARS

T. Angelov

Department of Astronomy, Mathematical Faculty, Studentski trg 16, 11000 Belgrade, Yugoslavia

(Received: February 19, 1998)

SUMMARY: An integrated distribution function is derived for visual double stars according to the magnitude difference Δm between the components. For this purpose the author uses a sample of 1626 double stars with $\Delta m \in [0^m - 4^m]$. The increment of the descriptive distribution function for an arbitrary increment of the variable Δm is also determined.

1. INTRODUCTION

The double-star statistics is a badly solvable task. There are no reliable indicators concerning the binary-system distribution in mass or orbital elements and also their spatial distribution does not differ from that of other stars of the galactic field. All of this imposes a conclusion that there are no "Double Star Populations", homogeneous in a selected characteristic (physical, kinematical or geometric). Of course, the division into visual, spectral, eclipse or close binaries is not the topic here.

On the other hand, the observational material concerning the double stars enables the deriving of empirical relations suitable for testing of theoretical star models (with or without rotation effects), as well as for examining the stability of stellar atmospheres. In this connexion the most ample application belongs to the 'mass–luminosity' relation (Harris *et al.*, 1963; McCluskey and Kondo, 1972; Popović and Angelov, 1972; de Jager, 1980; Angelov, 1993a; Angelov, 1993b). In view of the correlation ($\log L$, $\log \mathcal{M}$). ($\log \mathcal{M}$, M_b), as well as ($\log \mathcal{M}$, M_V), one should expect a statistical dependence between the magnitude difference and the mass relation of the double-star components.

In other words the distribution of double stars in magnitude difference of the components may serve as indicator of their distribution, for example, in the mass ratio of the components. In this paper one considers the possibility of deriving an integrated distribution function for visual double stars in the magnitude difference of the components. For this purpose the author uses the observational material (Zverev, 1979) on which the Belgrade Visual-Double-Star Catalogue (Sadžakov and Dačić, 1990) is based.

2. THE STRUCTURE OF OBSERVATIONAL MATERIAL

The statistical sample contains 1626 visual double stars with magnitude difference between the components within $0^m - 4^m$. Let d be the increment in the variable $\Delta m = m_2 - m_1$, and $N(x)$ the number of systems with $\Delta m \leq x$. For the purpose of calculating the relative change $\Delta N/N$ for an arbitrary value of d a new function will be defined:

$$F(x + d,\, x) = \frac{N(x + d)}{N(x)}. \qquad (1)$$

Since $(x + d)$ belongs to the actual interval of Δm, the highest value of variable x is $x_m = 4.0 - d$, $d \in [0.1, 4.0]$. The empirical dependence $F(x + d, x)$ is illustrated in Fig. 1. The lower limit of this interval is determined by $F(x + 0.1, x)$, $x_m = 3.9$, the left hand one by the values $F(d, 0)$, and $F_{max} = F(4, 0)$.

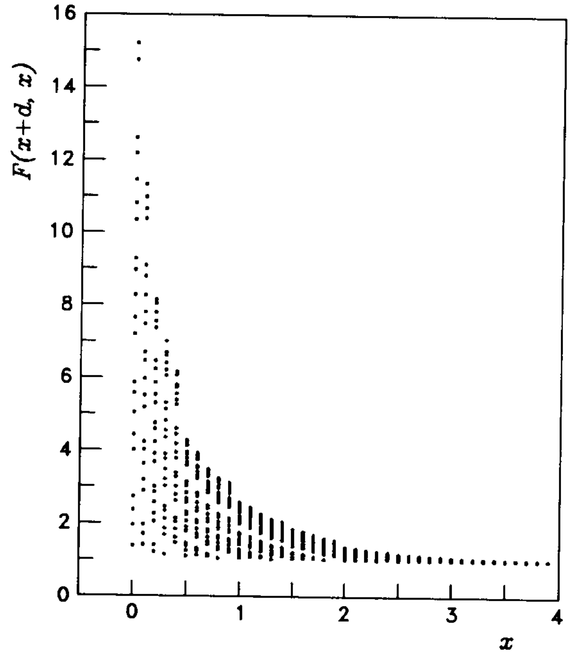

Fig. 1. *Empirical distribution $F(x + d, x)$ in the sample of 1626 visual double stars.*

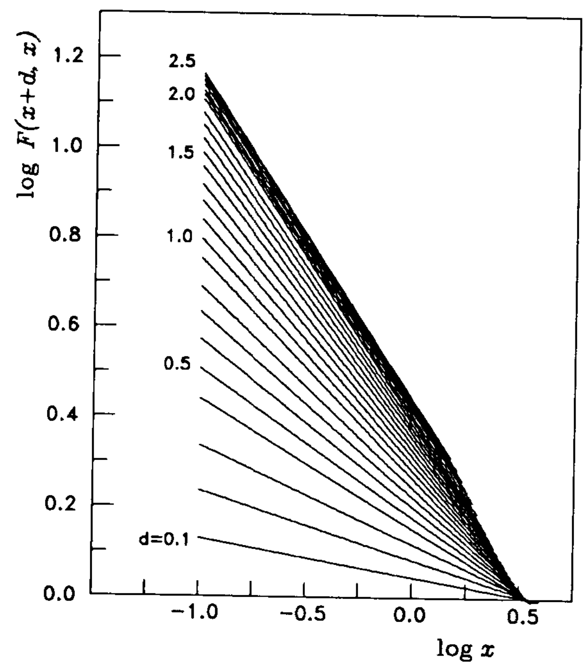

Fig. 2. *Linear correlation '$\log F(x + d, x)$ – $\log x$', $d \in [0.1, 2.5]$ — relation* (2).

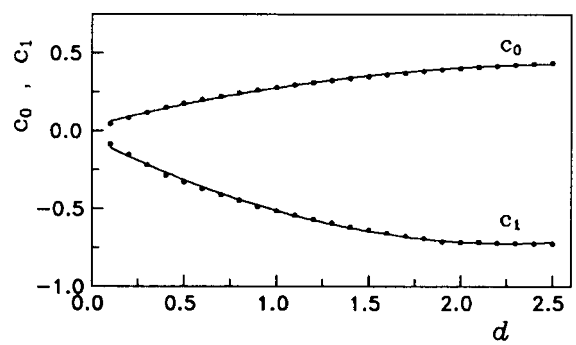

Fig. 3. *Coefficients $c_i(d)$ in relation* (2).

3. ANALYSIS AND CONCLUSION

For describing the empirical distribution F the following correlation is used:

$$\log F = \sum_{i=0}^{n} c_i (\log x)^i \,,$$

with coefficients c_i as functions of the increment d. Already at $n = 2$ a very good approximation is achieved. Here will be used the linear dependence:

$$\log F = c_0 + c_1 \log x \tag{2}$$

illustrated in Fig. 2 (the linear relation is less reliable with $d > 2.5$ though there is a trend of preserving the direction coefficients for $d = 2.5$).

The coefficients c_0 and c_1 can be represented as

$$c_i = \sum_{k=0}^{m} c_{ik} d^k \,, \quad i = 0, 1.$$

The c_i values are presented in Fig. 3, whereas the solid lines corresponding to a quadratic dependence $c_i(d)$:

$$\begin{aligned} c_0 &= 0.03 + 0.30\, d - 0.06\, d^2, \\ c_1 &= -0.05 - 0.60\, d + 0.13\, d^2. \end{aligned} \tag{3}$$

Now, relying on (2) we have:

$$F(x + d, x) = 10^{c_0} x^{c_1} \,, \qquad x, d \geq 0.1. \tag{4}$$

The interval of numerical values of this function (Fig. 4) is a description of the observed interval in Fig. 1 for $x \geq 0.1$.

The boundaries of distribution (4), i. e. boundaries of the region in Fig. 4 will be determined. The lower limit is the function $F(x + 0.1, x)$ with $c_i(0.1)$ from (3). Thus

$$F(x + 0.1, x) = 1.15\, x^{-0.11}. \tag{5}$$

This function determines the location of the empirical boundary in Fig. 1 with a relative error under 10 %.

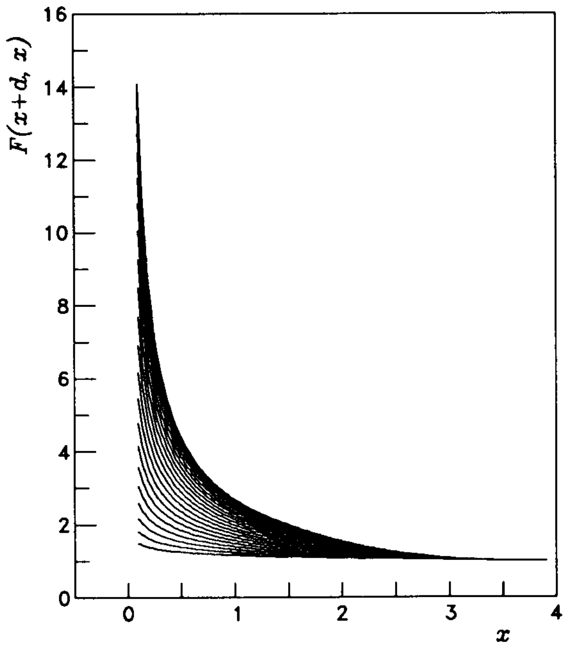

Fig. 4. *Descriptive distribution* $F(x+d,\, x)$ *for* $x \geq 0.1,\ d \leq 2.5$.

The left hand limit is specified by

$$F(0.1+d,\, 0.1) = 10^{c_0 - c_1}\,,$$

and according to (3)

$$c_0 - c_1 = a_0 + a_1 d - a_2 d^2,$$
$$a_0 = 0.08, \quad a_1 = 0.90, \quad a_2 = 0.19.$$

One has

$$F(0.1+d,\, 0.1) = A\,\exp\bigl[-\lambda(d-B)^2\bigr], \qquad (6)$$

with

$$A = \exp\left[\left(a_0 + \frac{a_1}{4a_2}\right)\ln 10\right] = 14,$$
$$B = \frac{a_1}{2a_2} = 2.4, \quad \lambda = a_2 \ln 10 = 0.44.$$

Unfortunately, the left hand limit is determined only for $d = 2.4$ (relative error of 2 % for $d = 0.1$ increases to 15 % for $d = 2.4$).

The upper boundary in Fig. 4 is specified by the values of function (4) for $x = x_m$, $d \leq 2.5$. In view of Fig. 2 and the correlation (2) for $d > 2.5$ it will be assumed that the boundary within $x_m < 1.4$ slightly deviates from its position in Fig. 4. In other words, the equation of the upper boundary line is approximatively

$$F(x_m + d,\, x_m) = \frac{N(4)}{N(x_m)}\,, \quad x_m \in [0.1,\, 3.9],$$

or according to (4):

$$N(x) = N(4)\cdot 10^{-c_0(d)} x^{-c_1(d)}, \quad d = 4 - x. \qquad (7)$$

On the other hand the boundary 'hyperbole' indicates the dependence

$$F^{-1}(x+d,\, x) = \sum_{i=0}^{j} b_i x^i\,,$$

yielding for $j = 2$:

$$\frac{N(x)}{N(4)} = 0.04 + 0.35\,x - 0.023\,x^2\,. \qquad (8)$$

With $N(4) = 1626$ the function $N(x)$ according to this relation, or according to (7) with c_0, c_1 from (3), is presented in Fig. 5. It is seen that the approximation of empirical $N(x)$ values is good for $x \geq 1$ (relative deviation is less than 2 %), but somewhat poorer for $x < 1$ (due to using $c_i(d)$ only $d = 2.5$, $x \leq 1.5$).

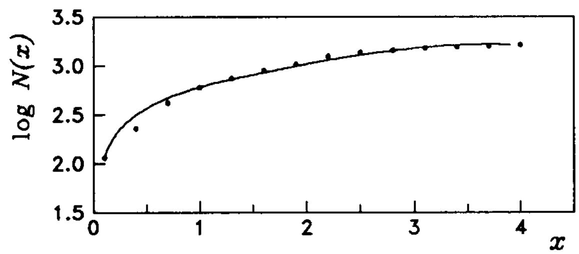

Fig. 5. *Integrated distribution function* $N(x)$ *for visual double stars with* $\Delta m \leq x \in [0.1,\, 3.9]$.

In any case the relation (4) enables the estimating of the number of visual double stars with $\Delta m \leq x + d$, $d = 4 - x$, from the observed number of these systems with $\Delta m \leq x$, $x \in [0.1,\, 3.9]$.

Acknowledgments – This work has been supported by Ministry for Science and Tehnology of Serbia through the project "Astrometrical, Astrodynamical and Astrophysical Researches". The author thanks his colleagues Mrs. Z. Cvetković and Mr. M. Dačić for giving him the access to the observational material.

REFERENCES

Angelov, T.: 1993a, *Publ. Obs. Astron. Belgrade*, **44**, 81.

Angelov, T.: 1993b, *Bull. Astron. Belgrade*, **148**, 1.

de Jager, C.: 1980, *The Brightest Stars* – D. Reidel Publ. Com., Dordrecht.

Harris, D.L., Strand, K.A. and Worley, C.E.: 1963, in: *Basic Astronomical Date*, ed. Strand, K. A., Chicago.

McCluskey, G.E., Kondo, Y.: 1972, *Astrophys. Space Sci.* **17**, 34.

Popović, G.M., Angelov, T.: 1972, *Bull. Obs. Astron. Belgrade*, **125**, 147.

Sadžakov, S.N., Dačić, M.D.: 1990, *Publ. Obs. Astron. Belgrade*, **38**.

Zverev, M.S.: 1979, *Private comunication.*

О РАСПОДЕЛИ ВИЗУЕЛНИХ ДВОЈНИХ ЗВЕЗДА

Т. Ангелов

Катедра за астрономију Математичког факултета, Студентски трг 16, 11000 Београд, Југославија

УДК 524.382
Оригинални научни рад

Изводи се интегрална функција расподеле визуелних двојних звезда у односу на разлику магнитуда компонената Δm. У том циљу користи се узорак од 1626 двојних звезда са $\Delta m \in [0^m - 4^m]$. Одређује се и прираштај описне функције расподеле за произвољан прираштај промењливе Δm.